8-1 Regular Expressions

re :=	$\emptyset$	Empty
	ε	Epsilon
	c	Char
	re $\cup$ re	Union
	re *	Star
	re $\circ$ re	Circ

$L : re \to P(\Sigma^*)$

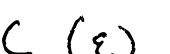
$C : re \to NFA$

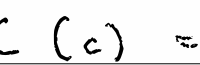
8-2 /

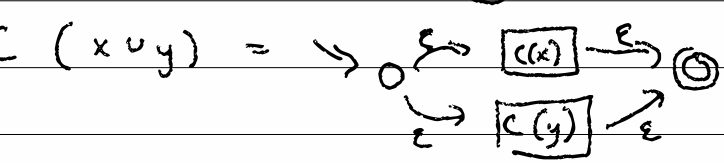
$$L(\emptyset) = \emptyset$$
$$L(\varepsilon) = \{\varepsilon\}$$
$$L(c) = \{c\}$$
$$L(x \cup y) = L(x) \cup L(y)$$
$$L(x^*) = L(x)^*$$
$$L(x \circ y) = L(x) \circ L(y)$$

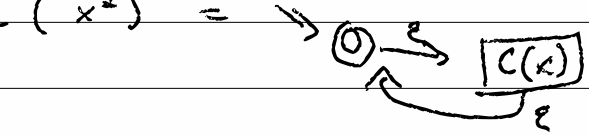
8-3) $C : re \rightarrow NFA$

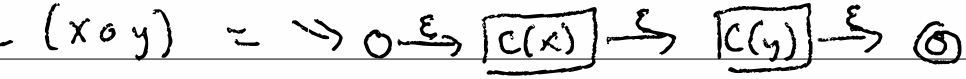
$C(\emptyset) =$ 

$C(\varepsilon) =$ 

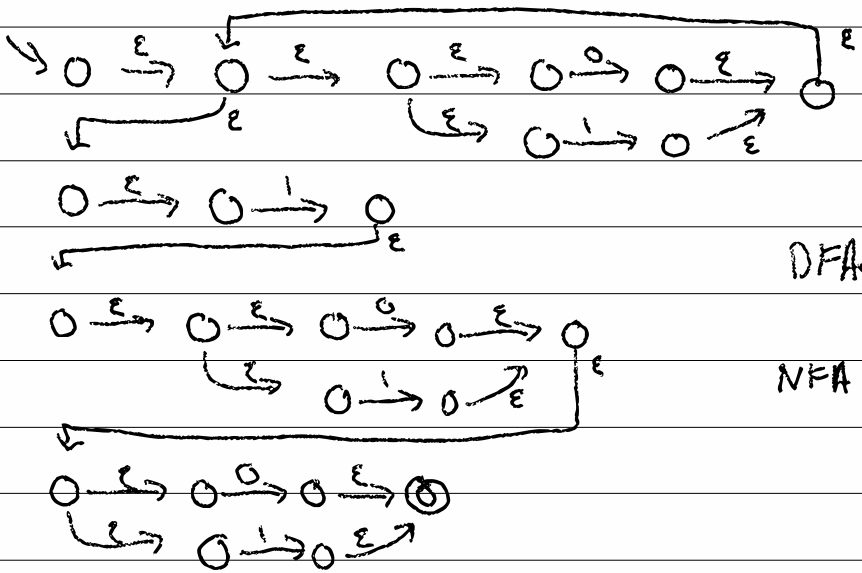
$C(c) =$ 

$C(x \cup y) =$ 

$C(x^*) =$ 

$C(x \circ y) =$ 

8-4/ $re = (0 \cup 1)^* \circ (1 \circ ((0 \cup 1) \circ (0 \cup 1)))$



DFAs - time $O(w)$
mem $O(\lg s + t)$

NFA - time $O(w)$
mem $O(n)$

8-5] interface Regex {}

class RE_Empty ()

class RE_Epsilon ()

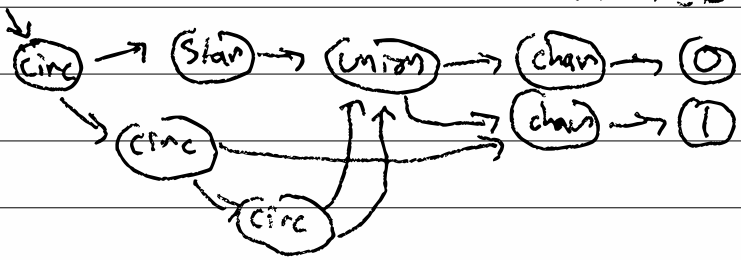
class RE_Char (char c)

class RE_Union (Regex lhs, Regex rhs)

class RE_Star (Regex arg)

class RE_Circ (Regex lhs, Regex rhs)

Ov1 = new RE_union (new RE_Char ('0'),
new RE_Char ('1'))



8-b) generate : re ⇒ str or false

gen $\emptyset$ = false

gen ε = ε

gen c = c

gen $(x \cup y)$ = let gx = gen x
if gx ≠ false then gx
O.w. gen y

gen (x^*) = gen $(\varepsilon \cup x \circ x^*)$

gen $(x \circ y)$ = let gx = gen x
let gy = gen y
if gx and gy ≠ false
gx ∘ gy
O.w. ret false

8-7

$\forall r \in re.$

$accepts_{dfa}(compile_{nfa}(compile_{re}(r)), generate(r)) = true$

8-8) $n + 0 = n$

$n \times 1 = n$

$a^* = \varepsilon \cup a \circ a^*$

$a \cup a = a$

$\emptyset \cup a = a = a \cup \emptyset$

$\varepsilon \circ a = a \circ \varepsilon = a$

$\emptyset \circ a = \emptyset = a \circ \emptyset$

$\emptyset^* = \varepsilon = \varepsilon \cup \emptyset \circ \emptyset^*$

$\varepsilon^* = \varepsilon = \varepsilon \cup \varepsilon \circ \varepsilon^*$

Michael Greenberg
Smart Constructors
are Smarter
than you think

$\delta_c\ \emptyset = \emptyset$

$\delta_c\ \varepsilon = \emptyset$

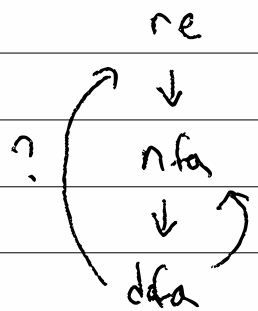
$\delta_c\ c' =$ if $c == c'$ then ε else $\emptyset$

$\delta_c\ (x \cup y) = \delta_c x \cup \delta_c y$

$\delta_c\ (x^*) = \delta_c x \circ x^*$

$\delta_c\ (x \circ y) =$ if $x \ni \varepsilon$ then $\delta_c x \circ y \cup \delta_c y$ else $\delta_c x \circ y$

8-9



Can I convert a DFA into a REX?