27-1 / $A_{TM} \notin \Sigma_0 \quad \in \Sigma_1$

$\overline{A_{TM}} \notin \Sigma_1 \quad \in ALL$

$\Sigma_1 < ALL$

Reducibility (mapping-)

A is reducible to B ($A \leq_m B$)

if $\exists f \in$ computable function.

$\forall w.\ w \in A$ iff $f(w) \in B$.

A = "decimal arithmetic" B = "binary arithmetic"

f("23 + 17 = 40") =

"10111 + 10001 = 101000"

27-2

$A \leq_m B$ and $B \in \Sigma_0$ then $A \in \Sigma_0$

$\in \Sigma_1$ $\in \Sigma_1$

$A \leq_m B$ and $A \notin \Sigma_0$ then $B \notin \Sigma_0$

$\notin \Sigma_1$ $\notin \Sigma_1$

$\forall B.\ A_{TM} \leq B \Rightarrow B \notin \Sigma_0 \wedge \overline{B} \notin \Sigma_1$

27-3) $E_{TM} \ni \langle M \rangle$ iff M is a TM
and $L(M) = \emptyset$

$A_{TM} \leq_m E_{TM} \iff \exists f.$ $w \in A_{TM}$ iff
$w = \langle M, x \rangle$ $f(w) \in E_{TM}$

M accepts x iff $L(M') = \emptyset$

$M'(y) =$ run M on x, if it accepts
then reject y.
O.w. accept y.

27-4/ $REG_{TM} \ni \langle M \rangle$ iff M is a TM
and $L(M) \in REG$

$f: \langle M, w \rangle \longrightarrow \langle M' \rangle$

M accepts w iff $L(M') \in REG$

$M'(x) =$ run M on w, if accepts return Yes
o.w. check if $x \in 0^n 1^n$

CFL_{TM}

27-5/ $EQ_{TM} \ni \langle M_1, M_2 \rangle$
iff $L(M_1) = L(M_2)$

$E_{TM} \leq_m EQ_{TM}$

$f : \langle M \rangle \rightarrow \langle M_1, M_2 \rangle$
$L(M) = \emptyset$ if $L(M_1) = L(M_2)$
$f(M) = \langle M, \rightarrow q_r \rangle$

27-6

Linear-Bounded Automata (LBA)

is a TM with no infinite tape

$\ell = (Q, \Sigma, \Gamma, q_0, \delta: Q \times \Gamma \to Q \times \Gamma \times \{L, R\}, q_a, q_r)$

$w \in L(\ell)$ iff $\sqcup [q_0] w \sqcup$

$0^n 1^n 0^n$ $\quad w \# w$ $\quad A_{DFA}$

27-7/ ALBA $\in \mathcal{E}_0$

$l = (Q, \Sigma, \Gamma, q_0, \delta, q_a, q_r)$

How many configs are possible?

If the input is w,

$$|Q| \times |w| \times |\Gamma|^{|w|}$$

wHw on a string that's 10 chars long

$$9 \times 10 \times 5^{10} =$$

878,906,250

<u>27-8</u>

$E_{LBA} \notin \cancel{\mathcal{D}} \Sigma_1$

$\overline{A_{TM}} \le_m E_{LBA}$

$f: \langle M, w\rangle \longrightarrow \langle M'\rangle$

where M (a TM) ~~accepts~~ does not accepts w

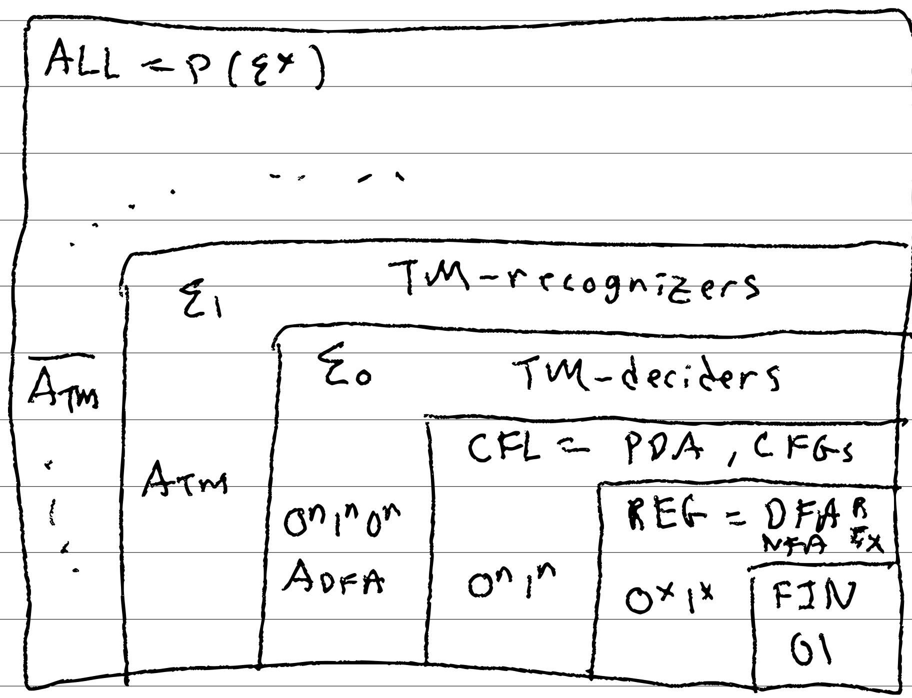
iff $L(M'$ (a LBA)$) = \emptyset$

$M'(x)$ = check that $x = c_0, c_1, c_2, \ldots c_n$

where $c_0 = e[M.q_0]w$ and $c_i \Rightarrow c_{i+1}$

$c_n = u[M.q_a]v$ by the TM rules

27-9



ALL = $P(\Sigma^*)$
Σ_1
TM-recognizers
$\overline{A_{TM}}$
Σ_0
TM-deciders
A_{TM}
CFL = PDA, CFGs
REG = DFA, NFA, REx
$0^n 1^n 0^n$
A_{DFA}
$0^n 1^n$
$0^* 1^*$
FIN
01