21-1 / TMs are closed under $\cup$

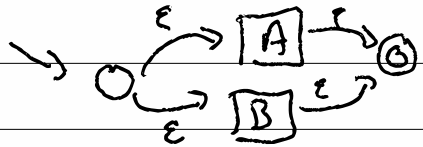
$\forall A, B.$

$A \in \mathcal{L}_0$ and $B \in \mathcal{L}_0$

$A \cup B \in \mathcal{L}_0$

CFGs: $C.S \Rightarrow A.S \mid B.S$

NFAs:



DFAs: $Q_C = Q_A \times Q_B$

$\delta_c((q_a, q_b), c) = (\delta_a(q_a, c), \delta_b(q_b, c))$

TM7=

21-2/ $(Q, \Sigma, \Gamma, \delta: Q \times \Gamma \to Q \times \Gamma \times \{L,R\}, q_0, q_a, q_r)$

Multiple Tape TM

$$\delta: Q \times \Gamma^k \to Q \times (\Gamma \times \{L,R\})^k$$

$k=2$

$$\begin{matrix} a \\ x \end{matrix}\left[\begin{matrix} q_i \end{matrix}\right]\begin{matrix} b \\ y \end{matrix} \qquad\qquad 011\left[\begin{matrix} \text{carry} \end{matrix}\right]\begin{matrix} 011 \\ 010 \end{matrix}$$

$$\delta(q_i, a, \alpha) = (q_j, (b,L), (\beta,R))$$

$$\begin{matrix} uc \\ x \end{matrix}\left[\begin{matrix} q_i \end{matrix}\right]\begin{matrix} av \\ \alpha y \end{matrix} \Rightarrow \begin{matrix} u \\ x\beta \end{matrix}\left[\begin{matrix} q_j \end{matrix}\right]\begin{matrix} cbv \\ y \end{matrix}$$

21-3/ $\cup : A \times B \to (A \cup B)$

TM $t_a = (Q_a, \Sigma, \Gamma_a, q_{0a}, \delta_a, q_{aa}, q_{ra})$

$\delta_a : Q_a \times \Gamma_a \to Q_a \times \Gamma_a \times \{L, R\}$

TM $t_b = (Q_b, \Sigma, \Gamma_b, q_{0b}, \delta_b, q_{ab}, q_{rb})$

$\delta_b : Q_b \times \Gamma_b \to Q_b \times \Gamma_b \times \{L, R\}$

MTM t_c: $Q_c = (Q_a \times Q_b) \cup \{q_a, q_r\}$

$\Gamma = \Gamma_a \cup \Gamma_b$ $\quad q_0 = (q_{0a}, q_{0b})$

$\delta((q_a, q_b), (c_a, c_b)) = (q_c, (c'_a, d_a), (c'_b, d_b))$

where $(q'_a, c'_a, d_a) = \delta_a(q_a, c_a)$

$(q'_b, c'_b, d_b) = \delta_b(q_b, c_b)$

$q_c = (q'_a, q'_b)$

$\delta((q_{aa}, \_), \_, \_) = (q_a, \_, \_)$

$\delta((\_, q_{ab}), \_, \_) = (q_a, \_, \_)$

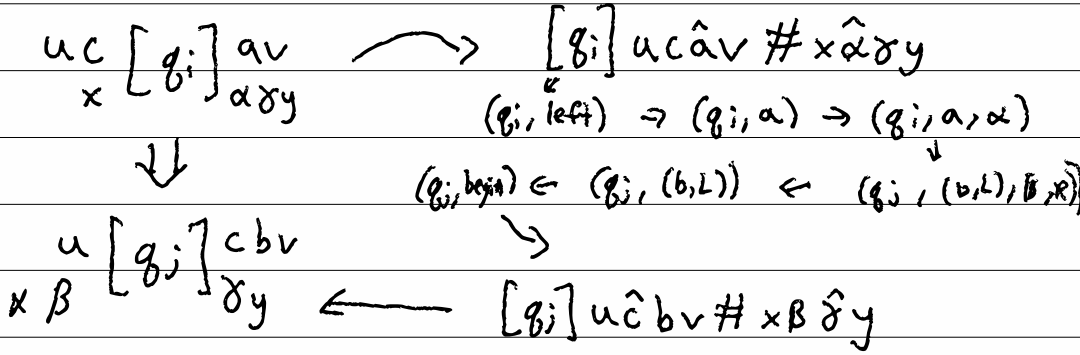
$\delta((q_{ra}, q_{rb}), \_, \_) = (q_r, \_, \_)$

21-4/ $\mathcal{E}_0$ is closed under $\cup / \cap$

$\mathcal{E}_1$ ....

$\Gamma' = \{0,1\} \times \Gamma \cup \{\#\}$

MTMs $\Longleftrightarrow$ TMs



21-5