20-1 / $(Q, \Sigma, \Gamma, q_0, \delta: Q \times \Gamma \to Q \times \Gamma \times \{L, R\}, q_a, q_r)$

"machines"   "languages/expression"

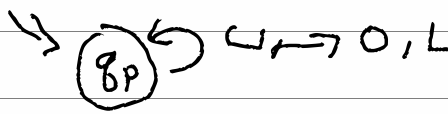
DFAs $\longleftrightarrow$ REXs

PDA $\longleftrightarrow$ CFGs

TM $\boxed{\text{enumerator}}$

$e = (Q, \Sigma, \Gamma, q_0, \delta: Q \times \Gamma \to Q \times \Gamma \times \{L, R\}, q_p)$

$w \in L(e)$ if $\varepsilon[q_0]\varepsilon \Rightarrow^* x[q_p]w$



0^*

20-2/ Suppose I had a Turing decider and I wanted an enumerator...

enumerator := try all strings in lexicographic and if the decider says yes, then it prints

recognizer $\Rightarrow$ enumerator := try all strings "in parallel"

enumerator $\Rightarrow$ recognizer := check the output...
never say no

enumerator "in order" $\Rightarrow$ decider := check the output...
say no, if something bigger printed

20-3/

DFAs $\longleftrightarrow$ REX

$\updownarrow$

NFAs (more expressive)

TMs $\longleftrightarrow$ Enumerators

$\updownarrow$

MT $\longleftrightarrow$ $\cup, \cap$

$\swarrow\!\!\nearrow$

NTM $\longleftrightarrow$ $\circ$

20-4/ want: an extension of Turing machines

$\forall a \in \text{input}.\ \exists b \in \text{output}.\quad M(a) = M(b)$

$$\begin{tikzcd} a \arrow[r, "\text{compile}"] \arrow[d, "m"] & b \arrow[dl, "m"] \\ x & \end{tikzcd}$$

Galois connection on a bisimulation

10-5

$\forall a \in \text{input}. \ \exists b \in \text{output}.$

$\text{eval}_i(a) \quad \text{similar} \quad \text{eval}_o(b)$

$$\begin{tikzcd}
A \arrow[r, "\text{compile}"] \arrow[d, "\text{eval}"'] & B \arrow[d, twoheadrightarrow, "\text{eval}"] \\
a & b \arrow[l, "\text{decompile}"]
\end{tikzcd}$$

20-b/ TMs w/ a "stay" action

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$$

$$\delta_S : Q \times \Gamma \to Q \times \Gamma \times \{L, R, S\}$$

$$\frac{\delta(q_i, a) = (q_j, b, S)}{u[q_i]av \Rightarrow u[q_j]bv}$$

20-7/ stay = input $\longrightarrow$ normal tm

$$\begin{array}{ccc} u[q_i]av & \xrightarrow{\text{compile}} & u[q_i]av \\ \downarrow & & \downarrow \\ & & ub[q_j^*]v \\ & & \downarrow \\ u[q_j]bv & \longleftarrow & u[q_j]bv \end{array}$$

$\delta(q_i, a) = (q_j, b, S)$

$\Rightarrow \delta(q_i, a) = (q_j^*, b, R)$

$\forall q_j \in Q,$ $\delta(q_j^*, \gamma) = (q_j, \gamma, L)$

$\gamma \in \Gamma.$