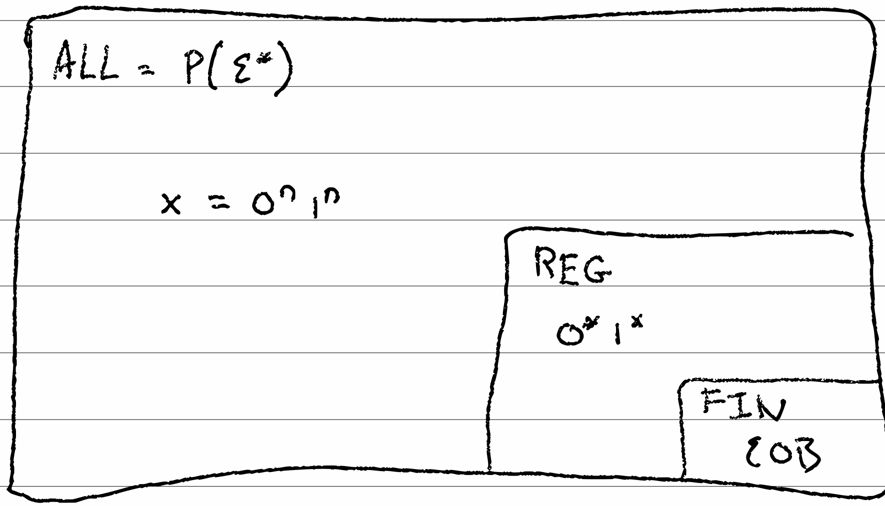
10-1



DFAs = NFAs = REXs = Regular Languages, REG

$REG \subseteq ALL$ $\quad$ $FIN \subseteq REG$

$REG \neq ALL$?

$\exists x \in ALL.\ \forall d \in DFA.\ L(d) \neq x$

10-2) principle of natural number induction

$$P\ 0$$

$$\forall n,\ P\ n \to P\ (n+1)$$

$$\to \forall n,\ P\ n$$

10-2) Goal: Prove that something is NOT in an infinite set ($0^n 1^n \notin REG$)

Strategy:

First, prove that all things in the set have some property

Next, prove that thing doesn't have that property.

Conclude, that thing is not in set.

0. What is P? ⟶ "regular pumping property"
1. Prove $\forall A \in REG.\ P(A)$ ⟶ "pumping lemma"
2. Prove $\neg P(x)$

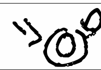
$\Rightarrow x \notin REG$

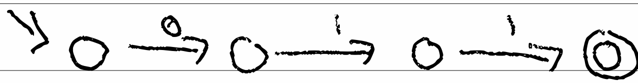
10-4) What is true about all regular languages?

There's some DFA that accepts them.

and there's one smallest DFA.

$011 \in A$





Every DFA has some number of states

say 4

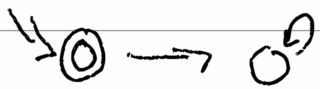
Suppose that the DFA accepts a string

of length 4 — 0110

10-5/ All DFAs have loops in them.

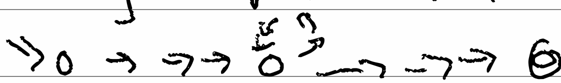
$$\delta : Q \times \Sigma \to Q$$

Some loops are boring ... once you reach them, everything is rejected



All finite languages have boring loops

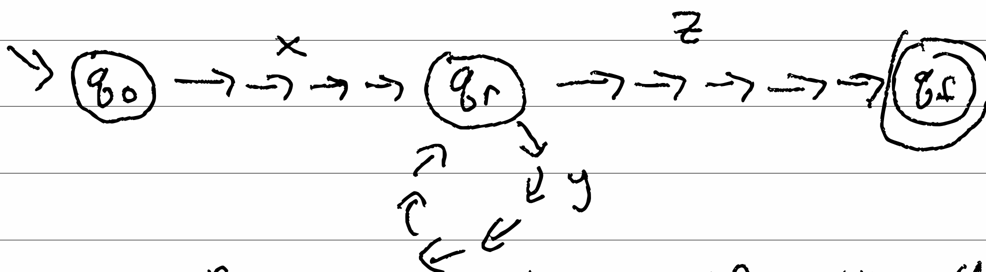
An exciting loop ... has a way to accept



10-6/ All regular languages have a DFA.

All DFAs that accept infinitely-many strings have exciting loops.

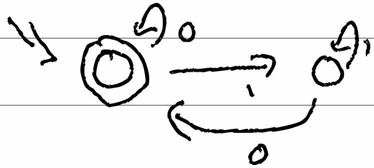
All regular languages that are infinite have "the same kind" of infinitely many strings.



If $xyz \in A$, then that $xz \in A$ $\quad xyyyz \in A$

$xyyz \in A$

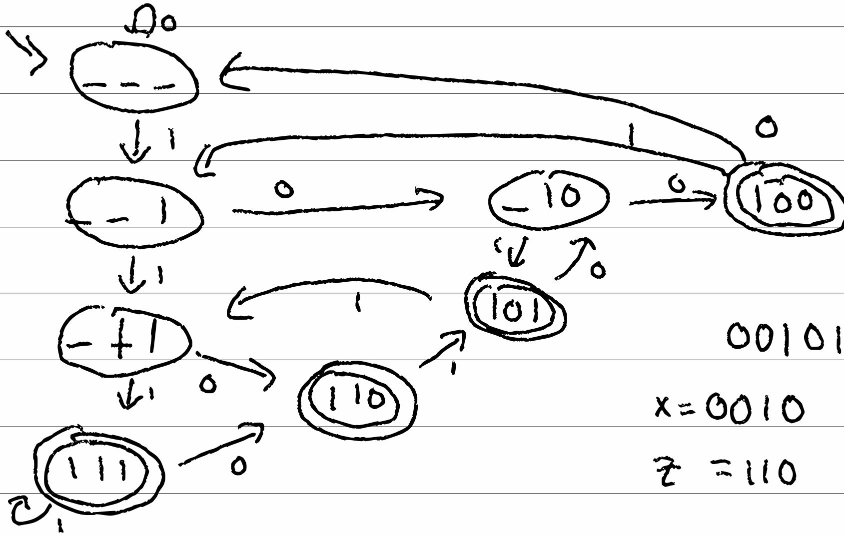
10-7) ends in 0 (even num)



$x = \varepsilon$

$y = 0$

$z = \varepsilon$



001010110

$x = 0010$ $y = 10$

$z = 110$

10-8) Regular Pumping Property (RPP)

$: P(\Sigma^*) \to \text{Prop}$

$RPP(A) :=$

$\exists p \in \mathbb{N},$

$\forall (s \in A \,.\, |s| \geq p)$

$\exists (x, y, z \in \Sigma^* \,.\, s = xyz$

$\wedge\ |xy| < p$

$\wedge\ |y| > 0)$

$\forall i \in \mathbb{N},$

$x \circ y^i \circ z \in A,$