

1-1 What computers can do ?

When is an algorithm correct?

What is a program?

What can math do?  
When is an equation true?  
Which sets really exist?

} mathematics  
foundations

1-2 A set is "bunch of stuff"

$\{ \text{pen, tablet} \}$       $\{ \checkmark, \square \}$

$\{ \text{tablet, pen} \}$

$\emptyset$  - empty set      $\{ \}$

pen  $\in \emptyset$  ? No.

$\forall x, x \in \emptyset$  is false

1-3 singleton set  $\{x\}$

$$\forall y. y \in \{x\} \text{ iff } y = x$$

union : set set  $\rightarrow$  set

$A \cup B$  is a set

$$\forall x. x \in A \cup B \text{ iff } x \in A \text{ or } x \in B$$

$$\{\text{pen}, \text{tablet}\} = \{\text{tablet}, \text{pen}\} = \{\text{pen}\} \cup \{\text{tablet}\} \\ \cup \emptyset$$

intesection =  $A \cap B$

$$\forall x. x \in A \cap B \text{ iff } x \in A \text{ and } x \in B$$

1-4 subset question

↓

$\subseteq$  : Set      Set  $\rightarrow$  Prop

$\forall A, B.$

$A \subseteq B$  iff  $\forall x. x \in A \rightarrow x \in B.$

$\{\text{pen}\} \subseteq \{\text{pen}, \text{tablet}\}$        $\emptyset \subseteq \{\text{pen}\}$

powerset       $P(A)$        $2^A$        $\emptyset, \{\text{pen}\}$

$\forall A.$

$\{\text{tablet}\}$

$\forall x. x \in P(A)$  iff  $x \subseteq A$        $\{\text{pen}, \text{tablet}\}$

if  $A$  has  $n$  elems, then  $P(A)$  has  $2^n$  elem

1-5 Complement  $A^c$   $\bar{A}$

$$\forall x, x \in A^c \text{ iff } x \notin A$$

Universe  $U$  is the set of things we  
talking about.

$$\forall x \in U, x \in \emptyset$$

1-6 Computations will be sets

... sets of strings

... and strings are sequences of characters

... and chars are elements of an alphabet

... an alphabet is a finite set

→ built with  $\cup, \emptyset, \epsilon$

$$\Sigma = \{0, 1\}$$

$$= \{ \text{0}, \text{1}, \text{0}, \text{1} \}$$

chars 0, 1

$\epsilon$  (epsilon) is an empty string ""

~~is a string~~  $cX$  is a string if  $c$  is a char  
and  $x$  is string

1-7	0ε	110ε	ε
	0	110	ε

$|x|$  length of string

$$|\epsilon| = 0$$

$$|cx| = 1 + |x|$$

A computation <sup>= problems</sup> is a set of strings

$\{\epsilon, 0, 00, 000, 0000, 00000, \dots\}$

"is a string made of only zeros?"

"are all the chars in a string zero?"

1-9

concatenation

$x \circ y$  is a string if  $x$  and  $y$  are strings

$$01 \circ 10 = 0110$$

Decide how to represent Alphabets and characters,

Alphabet = List < Character >

Char = Object

w/ equality and printing

Decide how to represent strings

String = List < Char >



L9 Every alphabet has a lexicographic ordering

$$\Sigma = \{0, 1\}$$

