9-1 / G: $\exists X \in ALL.\ X \notin REG$

1. $\forall A \in REG.\ P(A)$
2. $\neg P(X)$

$\implies X \notin REG$

$P = RPP$ (regular pumping property)

$P(A) :=$

$\exists p \in N.$

$\forall (w \in A \mid |w| \geq p)$

$\exists (x, y, z \in \Sigma^* \mid w = xyz \wedge |y| > 0 \wedge |xy| \leq p)$

$\forall i \in N.$

$xy^iz \in A$

$0^n1^n \notin REG \quad = \{w \in \{0,1\}^* \mid \exists n.\ w = 0^n1^n\}$

intuition: we need infinite states to count the 0s

$\neg RPP(A) :=$

$\forall p \in N$	Given p.
$\exists (w \in A \mid |w| \geq p)$	Choose w.
$\forall (x, y, z \in \Sigma^* \mid w = xyz \wedge |y| > 0 \wedge |xy| \leq p)$	Given x, y, and z.
$\exists i \in N.$	Choose i.
$xy^iz \notin A.$	Prove $xy^iz \notin A$.

$\neg RPP(0^n1^n)$

Given p. Choose $w = 0^p1^p$ or $0^{2p}1^{2p}$ or $0^{p/2}1^{p/2}$,

Given x, y, z, $w = xyz$, $|y| > 0$, $|xy| \leq p$.

$^i\circ c^j = c^{i+j}$

$w = xyz \qquad 0^p1^p = xyz \qquad x = 0^a \qquad y = 0^b \qquad z = 0^c1^p$

$a + b + c = p \qquad a + b \leq p \qquad b > 0$

$xy^iz \in A$ iff $0^a0^{b \cdot i}0^c1^p \in A$ iff $0^{a+bi+c}1^p \in A$ iff

$$\cancel{a} + bi + \cancel{c} = p$$
$$-(\cancel{a} + b + \cancel{c}) = -p$$
$$(i-1)b = 0 \text{ iff } i - 1 = 0 \text{ iff } i = 1$$

Choose $i = 0$.

$0^a0^c1^p \notin A$ easily because $a + b + c = p$ and $b > 0$

thus $a + c \neq p$

9-2

$\Sigma = \{0, 1\}$

$F = \{ ww \mid w \in \Sigma^* \}$ $\notin$ REG

$w = 0^p 0^p \in F$ $\geq p$

$x = 0^{p-2}$ $y = 00$ $z = 0^p$ $xy^iz \in F$ $\forall i$

$w = 0^p 1 0^p 1$

$D = \{ 0^{n^2} \mid n \in N \}$

$0 \in D$ $0000 \in D$ $0^9 \in D$

$w = 0^{p^2}$ $x = 0^a$ $y = 0^b$ $z = 0^c 0^{p^2-a-b-c}$

$0^{p^{2^{2^2}}}$

$G = \{ 0^n 1 0^m 1 0^{n+m} \mid n, m \in N \}$

$11 \in G$ "0+0=0"

$0101 00 \in G$ "1+1=2"

$w = 0^p 1 0 1 0^{p+1}$

$H = \{ 0^n 1 0^x \mid n \in N,\ x \text{ is the } n\text{th prime} \}$

$w = 0^p 1 0^x$ x is the pth prime

$\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +, =\}$

$G' = \{ x + y = z \mid \hat{x} + \hat{y} = \hat{z} \}$

$0+0=0 \in G'$ $12+24=36 \in G'$