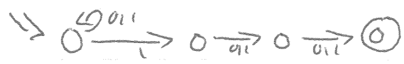
7-1

$C(r_1 \circ r_2) = \;\rightarrow \boxed{C(r_1)} \xrightarrow{\varepsilon} \boxed{C(r_2)} \xrightarrow{\varepsilon} ◎$

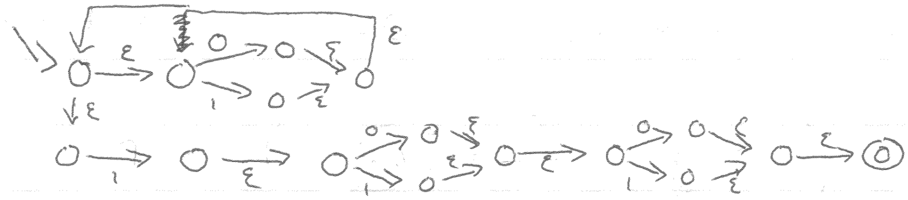
$\forall \underbrace{D \in DFA}_{ASM}.\ \exists \underbrace{R \in REX}_{PL}.\ L(R) = L(D)$ = This proof is like a decompiler

$(0 \cup 1)^* 1 (0 \cup 1)(0 \cup 1)$ ← 3rd from end is 1

by-hand NFA =



compiled NFA =



NFA / DFA $\xrightarrow{1}$ GNFA (n) $\xrightarrow{1}$ REX

Generalized NFA is $(Q, \Sigma, q_b, \Delta, q_e)$

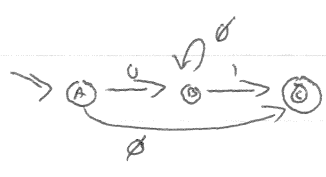
Q = the states $\quad q_b$ = the start state $\quad q_e$ = the end state

NFA: $\delta : Q \times \Sigma_\varepsilon \to P(Q)$ $\quad$ GNFA: $\Delta : \underbrace{(Q - q_e)}_{src} \times \underbrace{(Q - q_b)}_{dest} \to REX = \emptyset \mid \varepsilon \mid c \mid r_1 \circ r_2 \mid r_1 \cup r_2 \mid r_1^*$

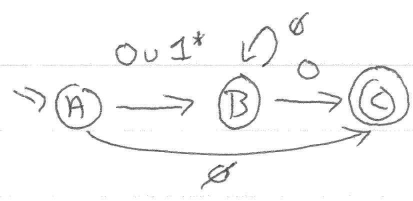
You can't leave q_e or return to q_b

$\Delta(q_i, q_f) = r$ iff $[q_i]\, x \circ w \Rightarrow^* [q_f]\, w$
$x \in L(r)$ (in an equivalent NFA (i.e. synthesizing δ))

$(q_i) \xrightarrow{r} (q_f)$



$\Delta(A, B) = 0$

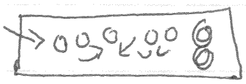
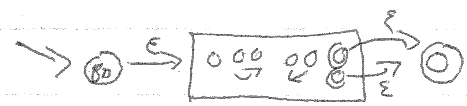


A k-GNFA is a GNFA with k states.
A k-NFA " "
A k-DFA " "

IN: k-NFA $\to$ (k+2)-GNFA
RIP: (k+1)-GNFA $\to$ k-GNFA
OUT: 2-GNFA $\to$ REX

DE := IN $\circ$ RIP^k $\circ$ OUT : k-NFA $\to$ REX

$OUT(\to q_b \xrightarrow{r} q_e) = r$

IN() = 

$\forall q_i \in Q, \forall q_j \in Q \quad \Delta(q_i, q_j) = c$ iff $q_j \in \delta(q_i, c)$
Q is for the NFA. $\quad\quad\quad\quad \emptyset$ o.w.

$\Delta(q_b, q_0) = \varepsilon \quad \Delta(q_b, q_i) = \emptyset$ for all $q_i \neq q_0$
$\Delta(q_f, q_e) = \varepsilon$ fo all $q_f \in F$ and $\Delta(q_i, q_e) = \emptyset$ for all $q_i \notin F$
$Q' = Q \cup \{q_b, q_e\}$

7-3)

RIP : $(k+1)$-GNFA $\rightarrow$ k-GNFA

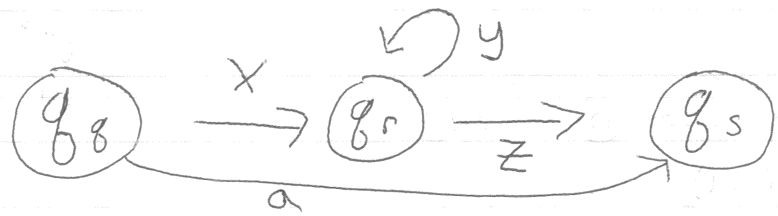
One state is removed!

input: $(Q, \Sigma, q_b, \Delta, q_e)$ $\qquad$ $Q = Q' \cup \{q_r\}$

contains q_b and q_e

output: $(Q', \Sigma, q_b, \Delta', q_e)$

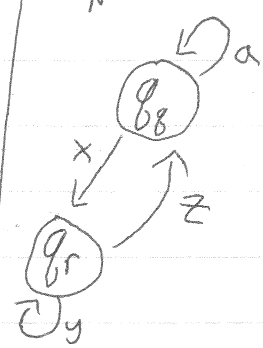
the victim, the ripped state



$\forall q_q \in \cancel{Q}\ (Q' - \{q_e\})\ \forall q_s \in \cancel{Q}\ (Q' - \{q_b\})$.

$\Delta(q_q, q_r) = x \qquad \Delta(q_r, q_r) = y \qquad \Delta(q_r, q_s) = z$

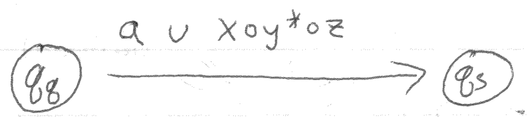
Special case $q_q = q_s$



$[q_q]\, uov \Rightarrow^* [q_s]\, v$ if$\cancel{f}$ $u \in L(xoy^*oz)$

$[q_q]\, uov \Rightarrow^* [q_s]\, v$ if$\cancel{f}$ $u \in L(a)$

new GNFA:



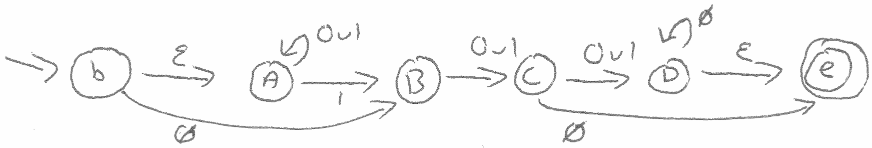
$\forall q_q \in Q - q_e,\ \forall q_s \in Q - q_b.$

$$\Delta'(q_q, q_s) = \Delta(q_q, q_s) \cup \Delta(q_q, q_r) \circ \Delta(q_r, q_r)^* \circ \Delta(q_r, q_s)$$

7Y/ 4N

$r^* = \varepsilon \cup r \circ r^*$

6G



5G

$\emptyset \cup \varepsilon \circ (0\cup 1)^* \circ 1$

$(0\cup 1)^* 1$

$\emptyset \cup x = x \cup \emptyset = x$

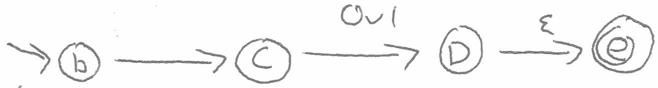
$\varepsilon \circ x = x \circ \varepsilon = x$

$\emptyset \circ x = x \circ \emptyset = \emptyset$

$\emptyset^* = \varepsilon$

$\varepsilon^* = \varepsilon$

4G



$\emptyset \cup ((0\cup 1)^* 1) \circ \emptyset^* \circ (0\cup 1)$

$= (0\cup 1)^* 1 (0\cup 1)$

3G

$(0\cup 1)^* 1 (0\cup 1)(0\cup 1)$

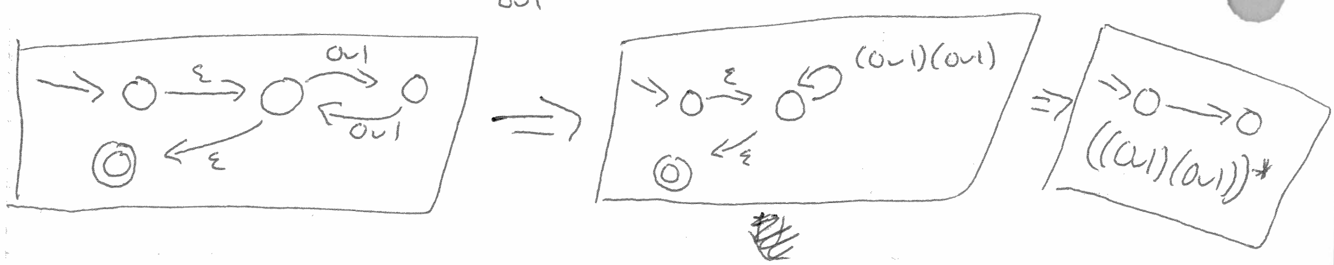
OUT

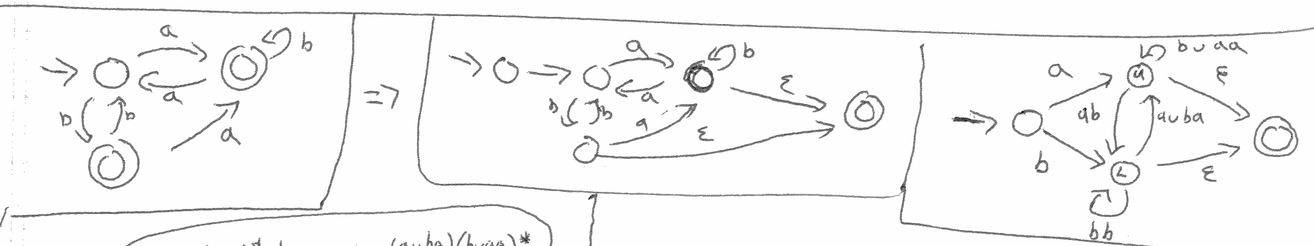
$(0\cup 1)^* 1 (0\cup 1)(0\cup 1)$

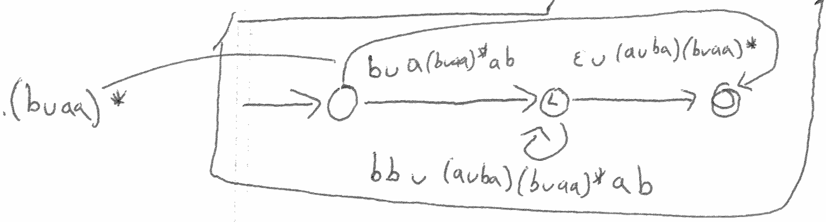
2G

Even strings

Is (??)*







$(b \cup a(b\cup aa)^* ab) \circ$

$((bb \cup (a\cup ba)(b\cup aa)^* ab))^* \circ$

$(\varepsilon \cup (a\cup ba)(b\cup aa)^*)$

$\cup\ (a(b\cup aa)^*)$