5-1

DFA $\dfrac{\delta(q_i, a) = q_j}{[q_i]\, aw \to [q_j]\, w}$

$q_i, q_j \in Q \quad w \in \Sigma^* \quad a \in \Sigma$

NFA $\dfrac{q_j \in \delta(q_i, a)}{[q_i]\, aw \to [q_j]\, w}$ --- $\in$

$q_i, q_j \in Q \quad w \in \Sigma^* \quad a \in \Sigma_\varepsilon$

Closure of REG under $\circ^k$, $A \circ B$, A^* --- $*$

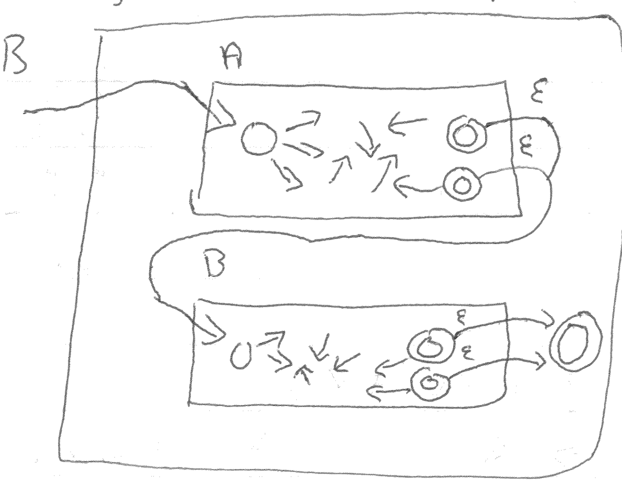
Kleene star

Closure of NFA under $\circ$:

$w \in A \circ B$ iff $w = x \circ y$ where $x \in A$ and $y \in B$

string set of string operation on sets op on strings

$A \circ B$



$A^0 = \varepsilon$

$A^1 = A$

$A^2 = A \circ A$

$A^{k+1} = A \circ A^k$

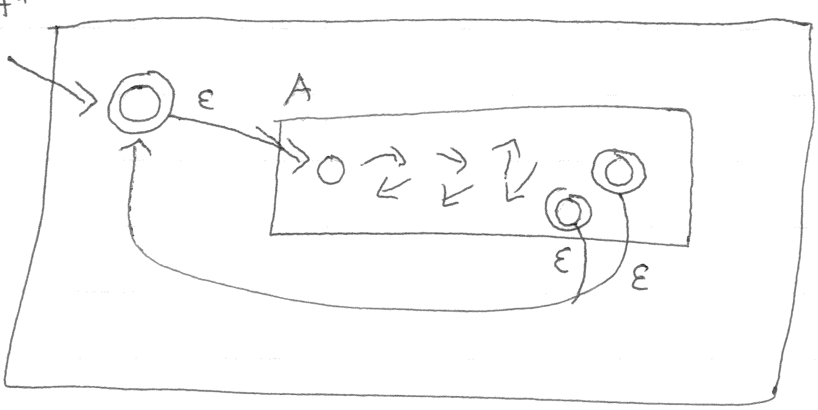
NFA closed under $*$

$w \in A^*$ iff $w = w_0 \ldots w_n$ and $\forall i,\ w_i \in A$

$w \in \varepsilon$ or $w \in A \circ A^*$

$A^* = \varepsilon \cup A \circ A^*$

A^*



5-2

DFA = NFA

$\Rightarrow$ $(\forall A.\ \exists D \in DFA \mid L(D)=A).\ \exists (N \in NFA).\ L(N)=A)$

$\Leftarrow$ $(\forall A.\ \exists N \in NFA \mid L(N)=A.\ \exists (D \in DFA).\ L(D)=A)$

$\Rightarrow$:

in: $D = (Q_D, \Sigma, q_{0D}, \delta_D, F_D)$

out: $N = (Q_N, \Sigma, q_{0N}, \delta_N, F_N)$

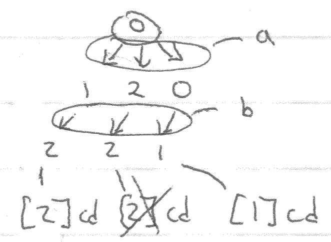
$Q_N = Q_D \quad q_{0N} = q_{0D} \quad F_N = F_D$

$\delta_N(q_i, a) = \{\delta_D(q_i, a)\}$

$\delta_N(q_i, \varepsilon) = \emptyset$

DFA diagrams are a subset of NFA diagrams

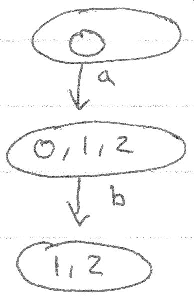
w=abcd



$\Leftarrow$:

in: $N = (Q_N, \Sigma, q_{0N}, \delta_N, F_N)$

out: $D = (Q_D, \Sigma, q_{0D}, \delta_D, F_D)$



$Q_D = P(Q_N)$

$q_{0D} = \{q_{0N}\}$

$F_D = \{q_i \in Q_D \mid q_i \cap F_N \neq \emptyset\}$

$\delta_D : Q_D \times \Sigma \rightarrow Q_D$

$\quad = P(Q_N) \times \Sigma \rightarrow P(Q_N)$

$\delta_N : Q_N \times \Sigma_\varepsilon \rightarrow P(Q_N)$

$\delta_D(\{q_0 \ldots q_m\}, c)$

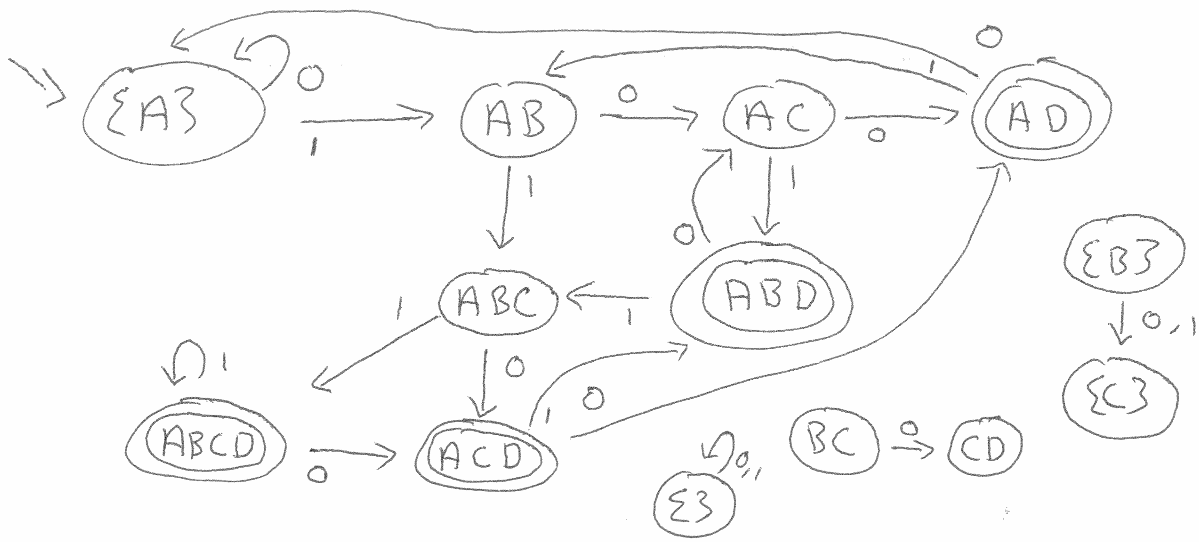
$\bigcup_{i \in [0,m]} \delta_N(q_i, c)$

That's all! *

5-3/ Third from end

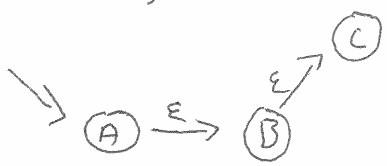
01100

⇓



If the NFA has N states, how many does the DFA have?
2^N

~~If the NFA~~ How much memory is used by the compiled DFA?
$\log(2^N)$ bits $= N$ bits



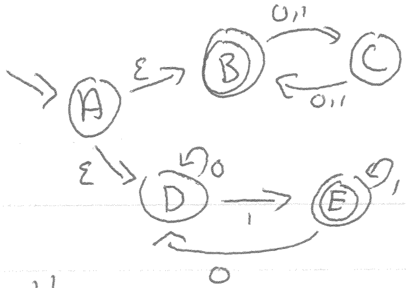
$q_{0N} = A$ $\quad$ $q_{0D} = \{A\}$ ✗
$= \{A, B, C\}$

OLD/BAD $\quad q_{0D} = \{q_{0N}\}$
NEW/GOOD $\quad q_{0D} = E(\{q_{0N}\})$ $\quad E : Q_D \to Q_D$
"epsilon closure"

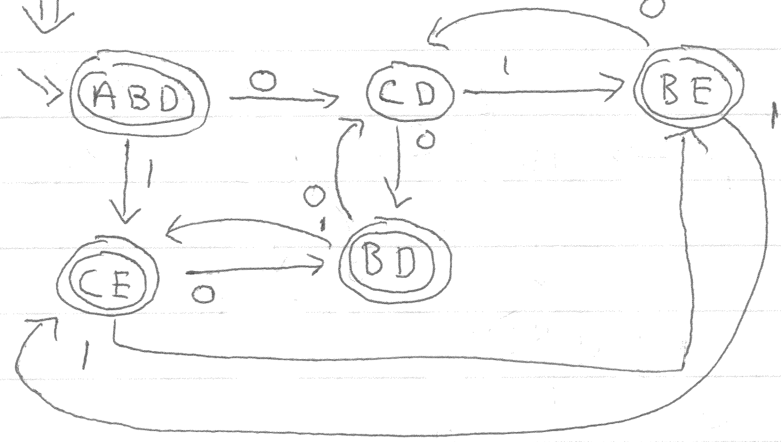
$q_i \in E(Q)$ iff $q_i \in Q$ or $q_j \in E(Q)$ and $\delta(q_j, \varepsilon) \ni q_i$

OLD: $\delta_D(q_D, c) = \underbrace{\bigcup_{q_i \in q_D} \delta_N(q_i, c)}$
NEW: $\quad = E(\qquad\qquad)$

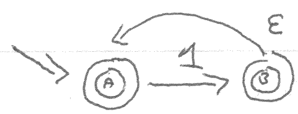
5-4

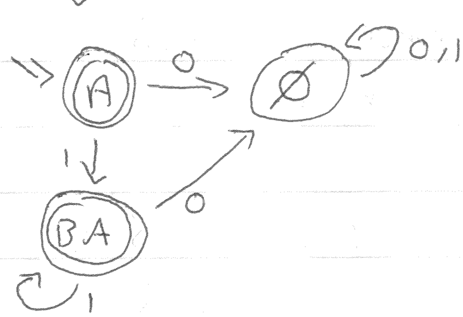


= Even OR ends in 1



Any number of 1s.





$w \in A^*$ iff $\exists k,\ w \in A^k$

$w \in A^k \longrightarrow w \in A^*$