4-1/

Repetition

"Ten even strings" → fixed # of repeats   "Five strings ending in 1"

"Any number of pokemon starter"

→ arbitrary number

Concatenation

"An even THEN a string ending in 1"

A^k — A k times A^* — A any number of times (including 0)

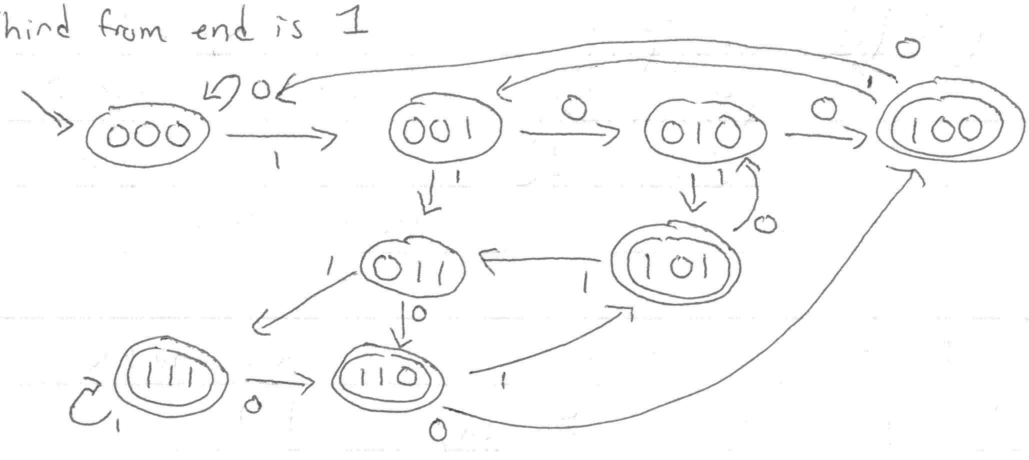
$A \circ B$ — A then B

"$('0' \cup '1')^* \circ '+' \circ ('0' \cup '1')^*$" $\ni$ 01 + 10

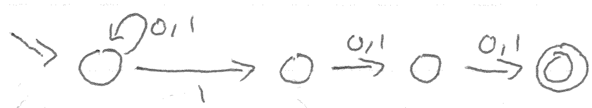
00111 + 010

Third from end is 1

DFA



NFA



Coming attractions!

NFA = DFA

Example

Intuition   0100

Formal def   could vs must

Formal semantics   NFA   DFA

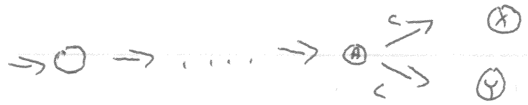
Properties   01000

4-2

NFA evaluation

Oracle - You know what choices are right and take them

Parallelism - whenever you have a choice, do both



— Spawn a 2nd thread to do Y, continue on X

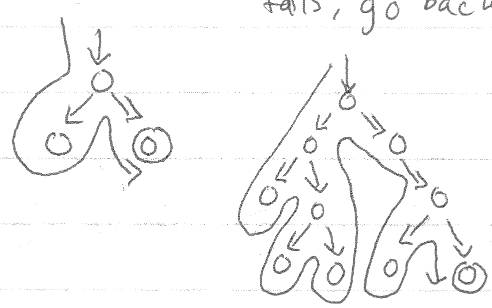
when string ends, OR statuses of all threads

if no choices, die

time - linear

space - $2^{|w|}$ - exponential in input

Backtracking - whenever you have a choice, try the left, if it fails, go back and try the right



(Depth-first search of ~~state~~ config space)

if all fail, say no

if one succ, say yes

time - exponential in input   space — same as DFA*

An NFA - non-deterministic finite automata

5-tuple $= (Q, \Sigma, q_0, \delta, F)$

Q, Σ, q_0, F are the same as with DFAs

1 thing

OLD DFA $\delta: Q \times \Sigma \rightarrow Q$

NEW NFA $\delta: Q \times \Sigma_\varepsilon \rightarrow P(Q)$

epsilon   subset of all the things

$$\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$$

$\delta(q_i, \varepsilon)$ = where to go from q_i if you don't want to read a character

4-3

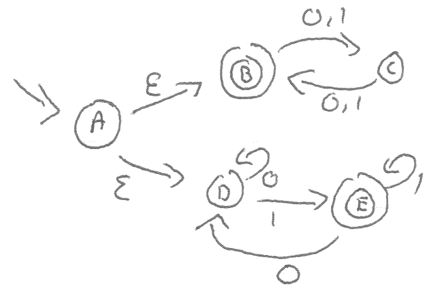
$E =$ 

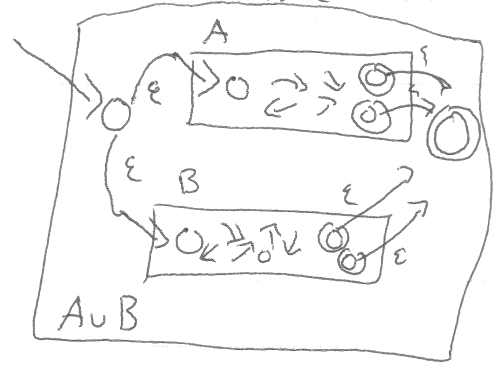
ends in 1 = 

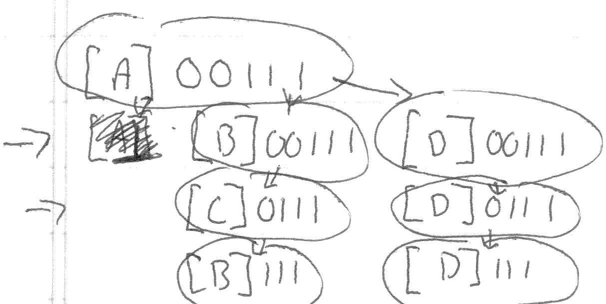
E;1

$E \cup E;1 =$ 

General argument that NFAs are closed under $\cup$

Also the union = 





in: $A = (Q_A, \Sigma, q_{0A}, \delta_A, F_A)$

$B = (Q_B, \Sigma, q_{0B}, \delta_B, F_B)$

out: $C = (Q_C, \Sigma, q_{0C}, \delta_C, F_C)$

$Q_C = \{S, E\} \cup (1 \times Q_A) \cup (Q_B \times 1)$

$q_{0C} = S \qquad F_C = \{E\}$

$\delta_C(S, \varepsilon) = \{(1, q_{0A}), (q_{0B}, 1)\}$

$\forall q_{fa} \in F_A, \ \delta_C((1, q_{fa}), \varepsilon) = \{E\}$

$q_{fb} \in F_B, \ \delta_C((q_{fB}, 1), \varepsilon) = \{E\}$

$(\forall q_a \in Q_A, \ \delta_C((1, q_a), t) = (1, \delta_A(q_a, t)) \ \forall t \in \Sigma)$