23-1

$0^n 1 0^m 1 0^{n \times m}$ — multiplication machine

//

$\langle n * m = o \rangle$ s.t. $o = n \times m$

$0^n 1 0^m 1 0^o$

$0^n * 0^m = 0^o$

n is bin × m in bin = o in binary

n in roman

n in arabic

---

$A_{DFA}$ := the "acceptance" problem on DFAs

$\langle D, w \rangle \in A_{DFA}$ iff w is accepted by the DFA D.

$A_{DFA}$ is DFA interperter

$\forall D \in DFA.\ \exists M \in TM.\ L(M) = L(D)$

$\exists$ DFA compiler

in: $(Q, \Sigma, q_0, \delta, F)$

out: $(Q', \Sigma, \Gamma, q_0', \delta', q_a, q_r)$

$Q' = Q \cup \{q_a, q_r\}$ $\qquad \Gamma = \Sigma \cup \{\sqcup\}$

$q_0' = q_0$

$\delta'(q_i, a) = (\delta(q_i, a), \sqcup, R) \qquad a \in \Sigma$

$\delta'(q_i, \sqcup) = q_a$ if $q_i \in F$

$q_r$ o.w.

tape 1: $\langle D, w \rangle = \langle (Q, \Sigma, q_0, \delta, F), w \rangle$

tape 2: $\langle q_0 \rangle \to \langle q_i \rangle \to \langle q_2 \rangle \to$

tape 3: w

---

Meta-program — a program about programs

interpreter : Prog ⇒ Ans

compiler : $Prog_A$ ⇒ $Prog_B$

analysis : Prog ⇒ Bool

generator : Num ⇒ Prog

23-2

$A_{NFA} = \{ \langle N, w \rangle \mid w \in L(N) \}$

$M_{A_{NFA}} = \text{COMPILER}_{NFA,DFA} \circ M_{A_{DFA}}$

$\text{Trans NFA DFA} = \{ \langle N, D \rangle \mid N \in NFA, D \in DFA, L(N) = L(D) \}$

$N \to D$ st. $L(D) = L(N)$

$A_{DFA}$ $\quad$ $A_{NFA}$ $\quad$ $A_{REX}$ $\quad$ $\in \Sigma_0$? $\quad$ $\in \Sigma_1$?

✓

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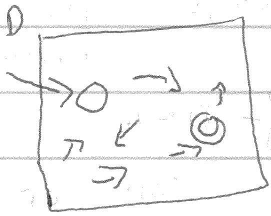
$E_{DFA}$ $\quad$ $\langle D \rangle \in E_{DFA}$ iff $L(D) = \emptyset$

D


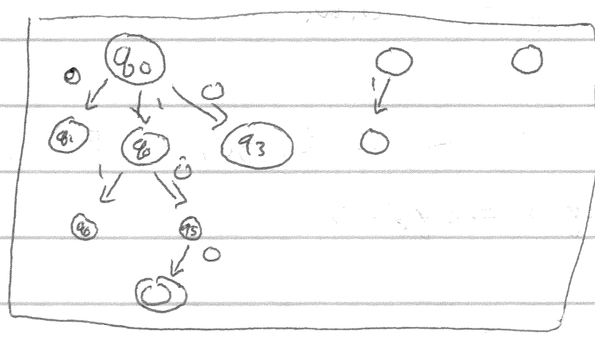
Is there a path from $q_0$ to something in F?

$V = Q$ $\quad$ $(x, y) \in E$ iff $\delta(x, a) = y$ for some a

What is the spanning from $q_0$? What is rachable from $q_0$?

$R(A, Q \cup \{q_i\}) = R(A \cup \{q_i\}, Q \cup \{q_j \mid \exists a. \delta(q_i, a) = q_j\} \setminus (A \cup \{q_i\}))$

$R(A, \emptyset) = A$ $\qquad$ $R(\emptyset, \{q_0\}) \cap F = \emptyset$?



23-3

$EQ_{DFA} = \{ \langle D_1, D_2 \rangle \mid D_x \in DFA \text{ and } L(D_1) = L(D_2) \}$

$E_{DFA}(\langle C \rangle) = Y$ iff $L(D_1) = L(D_2)$

$C = D_1 \cap D_2$ — nothing in common — want everything

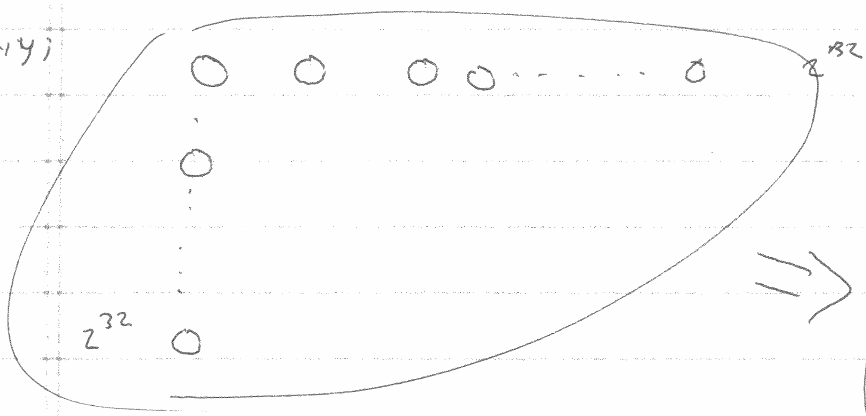
$C = \overline{D_1} \cap D_2$ — stuff that $D_2$ does but $D_1$ doesn't — want empty

$C = (\overline{D_1} \cap D_2) \cup (D_1 \cap \overline{D_2})$ — all mismatches $\rightarrow$ want empty

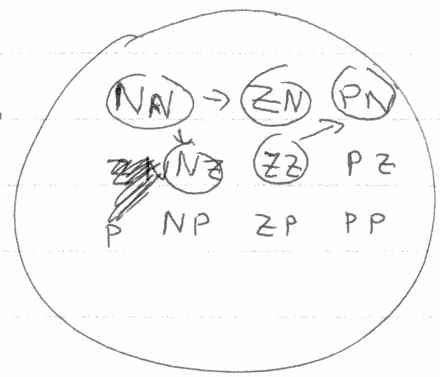
$(N \times N) \times (N \times N) = N^4$

Model Checking

int x, y;




$\Rightarrow$




$\hat{\delta}(\hat{q}_i, a) = \hat{q}_j$ if $q_i \in \hat{q}_i$, $q_j \in \delta(q_i, a)$

$(2,3) \rightarrow (3,5)$

$\downarrow$

$(0,1)$