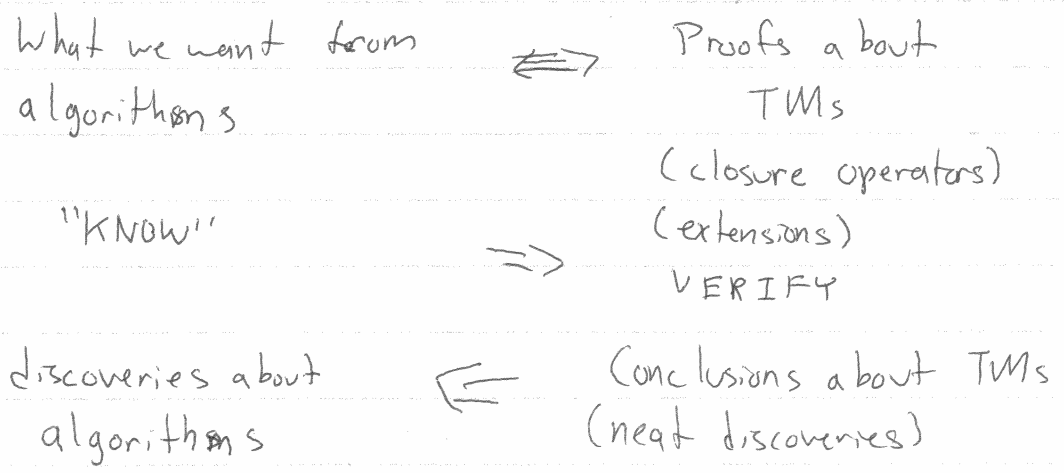


# Church-Turing Thesis

"algorithm" = "computation"  
 = "process"  
 " " " "  
 "software" = "hardware"  
 " " " "  
 X-calculus = Turing-Machine  
 " " " "  
 Java = X86  
 = C ARM =  
 = Racket MIPS =

If  $X = Y$  and  $\forall x \in X. \neg P(x)$  then  $\forall y \in Y. \neg P(y)$   
 "algorithm" = TM  
 TM cannot A  
 then "algorithms" cannot A



# 22-2 / Hilbert's 10th Problem

In 1900, David Hilbert

"A process involving finitely many steps exists to test whether a polynomial has an integral root."

Polynomial = ~~variables~~ bunch of variables, you have an exponent and a coefficient, you add together

$$6x^3y^2z^2 + 3xy^2z - x^3 - 10$$

$(1, 2, 0) = 3$   
 $(3, 1, 2) = 6$

variables = x, y, z

$(3, 0, 0) = -1$

$(0, 0, 0) = -10$

integral means "an integer"

$$ax^2 + bx + c$$

$$K=3 \quad c_{\max} = \max(a, b, c) \quad c_1 = a$$

X-poly is a polynomial 1-variable

Matiyasevic discovered that the root is bounded by

$$\pm K \cdot \frac{c_{\max}}{c_1}$$

$K = \#$  of  $\neq 0$  coefficients

$c_{\max} =$  largest coefficient

$c_1 =$  coefficient of biggest exponent

$$[-18, 18]$$

No bound could exist for multivariable polynomials

we will show that some problems have no TM, therefore no algorithm (via C-T thesis)

$$\Sigma_0 : U, n, 0, *, ^, c$$

$$\Sigma_1 : U, n, 0, *, ^ \quad (\text{NOT under: } c)$$