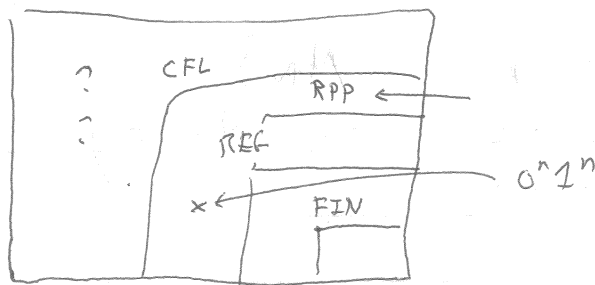


ALL



CFGs  $\leftrightarrow$  PDA  $\vdash$  CFL

REG  $\leftrightarrow$  DFA  $\vdash$  REG

CFL = ALL?

0. P : Language  $\rightarrow$  Property

REG  $\neq$  ALL  $\Leftarrow$  1.  $\forall A \in \text{REG. } P(A)$

2.  $\neg P(x)$

$\Rightarrow x \notin \text{REG}$

same strategy for CFLs

For DFAs, the finite # of states  $\rightarrow$  loop in states for long strings

CFG  $g = (V, \Sigma, R, S)$

only option string like  $\delta \in V$

$V = \{A\}$

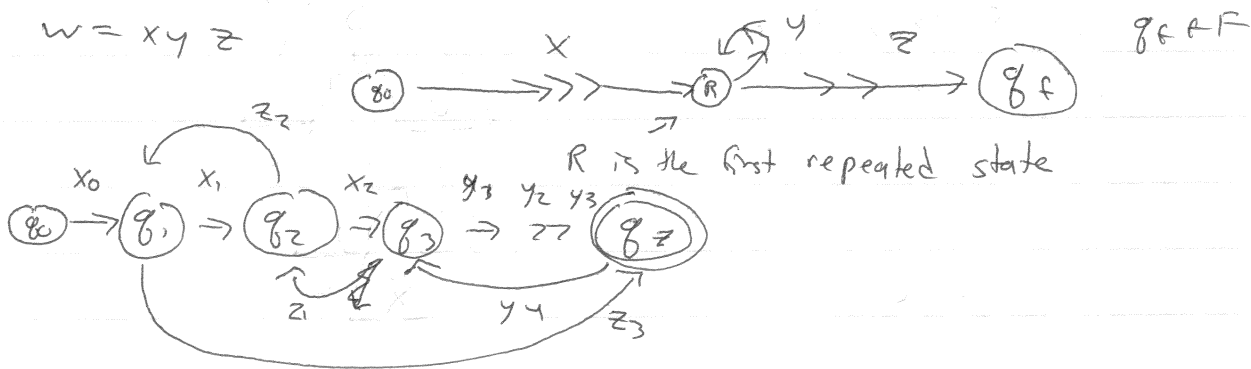
" $A \rightarrow \epsilon$ "

" $A \rightarrow a$ "

" $A \rightarrow AA$ "

Recall for DFAs,  $w \in A$  and  $|w| > |Q|$

$w = xyz$



15-2

Imagine CFG  $G = (V, \Sigma, R, S)$  has  $N$  variables

$S = A_1$        $V = \{ A_1, \dots, A_N \}$

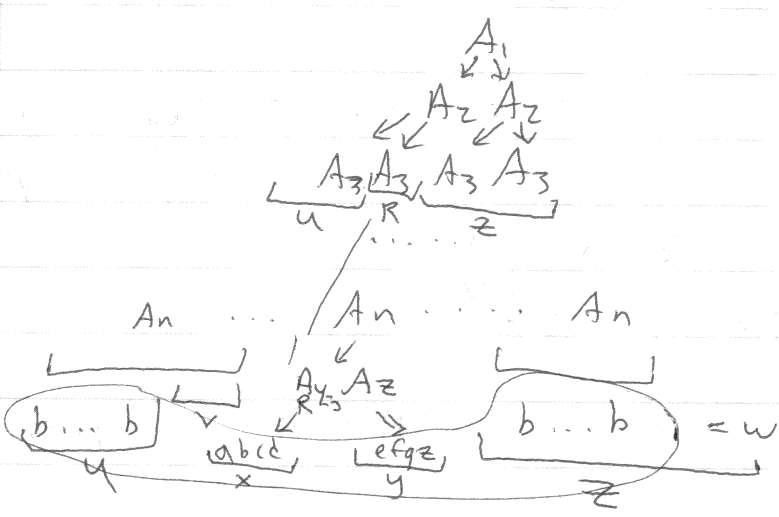
Assume in CNF  $( A_1 \rightarrow \epsilon \quad A_x \rightarrow A_y A_z )$   
 $( A_x \rightarrow a )$

$A_1 \rightarrow A_2 A_2$

$A_2 \rightarrow A_3 A_3$

$A_i \rightarrow A_{i+1} A_{i+1}$

$A_n \rightarrow A_y A_z$        $A_y$  and  $A_z$  have been seen before

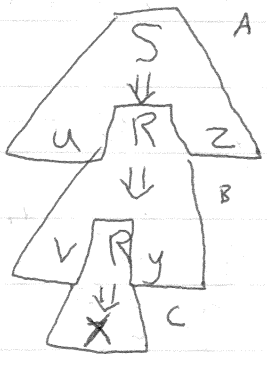


$2^n - 1$

$A_n \rightarrow b$

If  $|w| \geq 2^{n-1} - 1$ , then a variable is repeated!

- A.  $S \rightarrow u R z$
- B.  $R \rightarrow v R y$
- C.  $R \rightarrow x$



ABBC

u v v x y y z

15-3/

$$S \rightarrow OS1/\epsilon \Rightarrow S' \rightarrow S \Rightarrow S' \rightarrow \epsilon / S$$

$$S \rightarrow OS1/\epsilon \Rightarrow S \rightarrow OS1/01$$

$$\Rightarrow S' \rightarrow \epsilon / OS1 / 01 \Rightarrow S' \rightarrow \epsilon / XB / AB$$

$$S \rightarrow OS1 / 01 \Rightarrow S \rightarrow XB / AB$$

$$\begin{aligned} X &\rightarrow AS \\ A &\rightarrow 0 \\ B &\rightarrow 1 \end{aligned}$$

$$w \in A^* \quad |w| > 2^5$$

$$0^{16} 1^{16} \quad 0^0 1^0 = \textcircled{1} \quad 0^1 1^1 = \textcircled{3}$$

$$0^2 1^2 = \textcircled{2} \textcircled{6} \textcircled{5} \quad S' \rightarrow XB \rightarrow ASB \rightarrow AABBB$$

$$0^3 1^3 = S' \rightarrow XB \rightarrow ASB \rightarrow AXBBB \rightarrow AASBBB \rightarrow AAABBBB$$

$$\begin{cases} S \rightarrow \epsilon X 1 \\ X \rightarrow 0 X 1 \\ X \rightarrow 001 \end{cases} \quad \begin{matrix} u = \epsilon & z = 1 \\ v = 0 & y = 1 \\ x = 001 \end{matrix}$$

$$u v^i x y^i z \in A \quad \forall i$$

$$0^i 001 1^i 1 \in A \quad 0^{2+i} 1^{2+i} \in 0^n 1^n \quad \forall i$$

Context-Free Pumping Property (A)

$$\exists p \in \mathbb{N}$$

$$\forall (w \in A \mid |w| \geq p)$$

$$\exists (u, v, x, y, z \in \Sigma^* \mid \begin{matrix} w = uvxyz \\ 0 < |vy| \\ |vxy| \leq p \end{matrix})$$

$$\forall i \in \mathbb{N}$$

$$u v^i x y^i z \in A$$

$$\forall A \in CFL$$


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$$CFPP(A)$$


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$$(A \in CFL)$$

$$\Leftrightarrow (\exists G \in CFG. L(G) = A)$$

$$\forall A \in ALL$$

$$(\exists G \in CFG. L(G) = A)$$

$$\Rightarrow CFPP(A)$$

*[Faint, illegible handwriting on lined paper]*