1-1

CFG

$S \rightarrow aTbUcV \mid Sb \mid \varepsilon$

$T \rightarrow V \mid Sab \mid \varepsilon$

$U \rightarrow c$

$V \rightarrow \varepsilon \mid ST$

Chomsky-Normal Form

All rules must be one of three categories:

	$S \rightarrow \varepsilon$	(start to epsilon)
(B, C are not S)	$A \rightarrow BC$	(var to two vars)
	$A \rightarrow a$	(var to terminal)

$S \rightarrow 0S1 \mid \varepsilon$ — step 1: add a new start var

$S' \rightarrow S$ — step 2: remove ε-rules

$S \rightarrow 0S1 \mid \varepsilon$

$S' \rightarrow \varepsilon \mid S$ — step 3: remove unit rules $(A \rightarrow B)$

$S \rightarrow 0S1 \mid 01$

$S' \rightarrow \varepsilon \mid 0S1 \mid 01$ — step 4: introduce intermediate vars and terminal vars

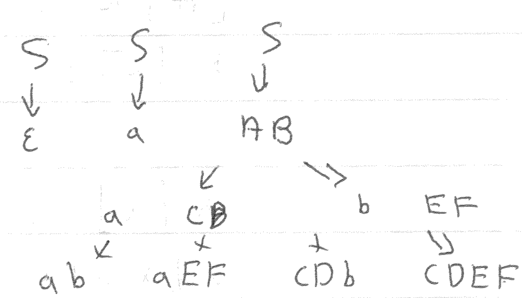
$S \rightarrow 0S1 \mid 01$

$S' \rightarrow \varepsilon \mid XB \mid AB$

$S \rightarrow XB \mid AB$

$X \rightarrow AS$

$A \rightarrow 0 \qquad B \rightarrow 1$

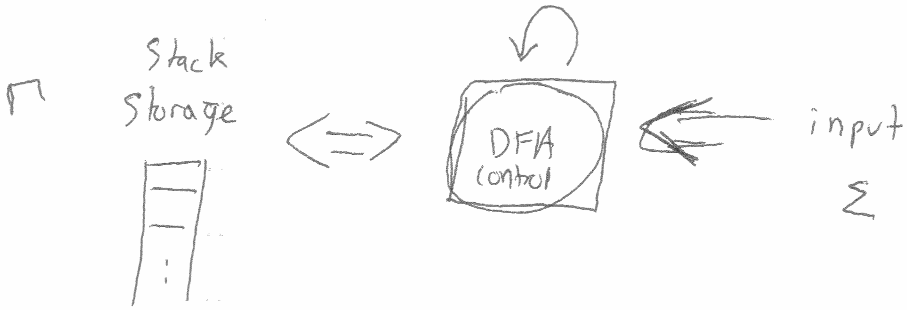


$\boxed{\text{PDA}}$: CFGs

DFAs : REX

11-2

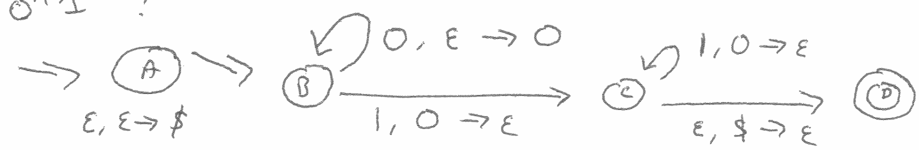
PDA (push down automata) recognizes for CFLs
(i.e. a machine for accepting strings in language)



Stack — look at top
push new top
(push / pop)

$\Sigma = \{0,1\}$
$\Gamma = \{0,\$\}$

$0^n 1^n$:



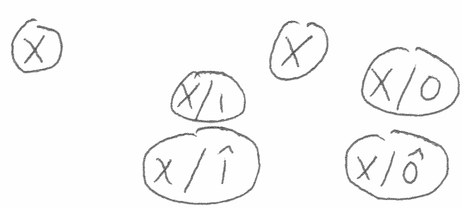
$x \xrightarrow[a,b \to \epsilon]{} y$ if input is a in state x and stack top is b (pop off) then go to y and push c

DFAs config: $[q]\,w$ where $q \in Q$ $w \in \Sigma^*$
PDAs config: $u\,[q]\,v$ where $q \in Q$ $v \in \Sigma^*$ $u \in \Gamma^*$
initial config is $\varepsilon[q_0]w$

$[A]0011 \to \$[B]0011 \to \$0[B]011 \to \$00[B]11 \to$
$\$0[C]1 \to \$[C] \to [D] \to$ YES

$X \xrightarrow{u,\ ?a \to ?c} Y$

$ab\,[X]\,uv$
$a\,[X/b]\,uv$
$c\,[X/\hat{b}]\,v$
$cb\,[X]\,v$



11-3

A PDA $p = (Q, \Sigma, \Gamma, q_0, \delta, F)$

Q is a finite set

Γ (stack) alphabet

Σ (input) is alphabet

$q_0 \in Q$

$F \subset Q$

$\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \to P(Q \times \Gamma_\varepsilon)$

$$\frac{(q_j, c) \in \delta(q_i, a, b)}{vb\,[q_i]\,ax \to vc\,[q_j]\,x}$$

$q_i, q_j \in Q$

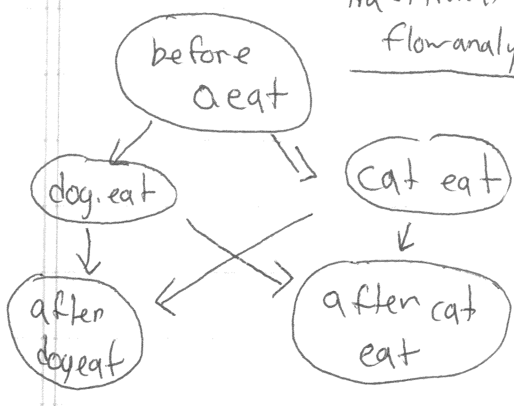
$a \in \Sigma_\varepsilon \quad b, c \in \Gamma_\varepsilon$

$x \in \Sigma^* \quad v \in \Gamma^*$

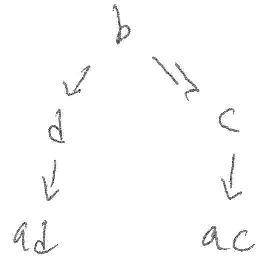
$[q_0]\,w \to^* v\,[q_f]\,\varepsilon$ where $q_f \in F$

iff $w \in L(p)$

traditional flowanalysis



pushdown flow analysis



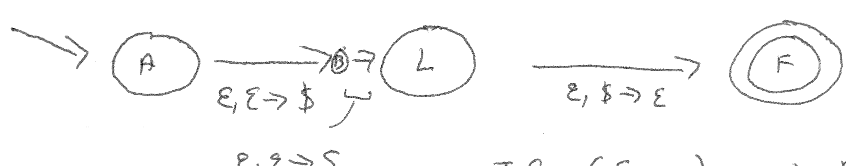
CFG $\Rightarrow$ PDA (Assume in C.N.F.)

input: (V, Σ, R, S)

$\Gamma = \Sigma \cup \cancel{V} \cup \{\$\}$

output: $(Q, \Sigma, \Gamma, q_0, \delta, F)$

$L/A_1 \;\cdots\; L(A_n)$ for all $A_i \in V$



If (S, ε) is in R, then $\delta(L, \varepsilon, S) = \{(L, \varepsilon)\}$

$(A, a) \in R$, then $L \to L$ on $\varepsilon, A \to a$

$(A, BC) \in R$, then $L \to L/B$ on $\varepsilon, A \to C$ and $L/B \to L$ on $\varepsilon, \varepsilon \to B$

If $a \in \Sigma$, then $L \to L$ on $a, a \to \varepsilon$

11-4/ [A] 0011 → \$ [B] 0011 → \$S' [L] 0011

\$B [L/X] 0011 → \$BX [L] 0011 → \$BS [L/A] 0011

\$BSA [L] 0011 → \$BSO [L] 0011 → \$BS [L] 011

→ \$BB [L/A] 011 → \$BBA [L] 011 → \$BB~~0~~ [L] 011

→ \$BB [L] 11 → \$B1 [L] ~~11~~ → \$B [L] 1 → \$1 [L] 1

→ \$ [L] → [F]