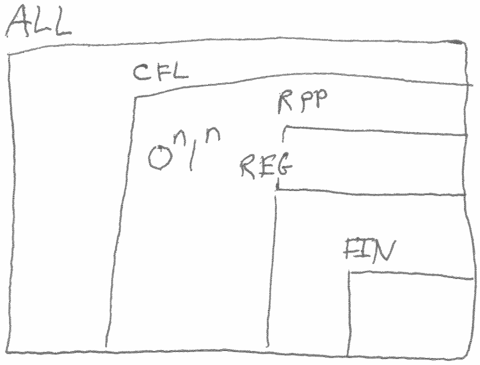
10-1



Category	machine	PL
REG	DFA	REX
CFL	PDA ②	CFG ①

CFGs - or Context-Free Grammars - define a new category the Context-Free Language

$$\boxed{\begin{array}{l} S \to \varepsilon \\ S \to 0S\underline{1} \end{array}}$$ is a CFG for 0^n1^n

$V \to w$ is called a rule, reduction, transition, or a substitution

S is a variable S is <u>the</u> start variable

0 and 1 are terminals

00<u>11</u> is in 0^n1^n? $S \to 0S1 \Rightarrow 00S11 \to 00\varepsilon 11 = 0011$

Tree → Tree Tree | ∙

$S \to S ; S \mid \text{return } E; \mid \text{if } E\ S\ S$

$E \to \underset{A}{E + E} \mid \underset{B}{E \times E} \mid \underset{C}{0} \mid \underset{D}{1} \mid E < E$

if 0 < 1 return 1 × 1 + 1; return 1 + 1 + 1;

1 × 1 + 1 A B D D D "1×1+1" is ambiguous

(A (B D D) D) (B D (A D D)) parse tree

10-2

A CFG g is a 4-tuple of (V, Σ, R, S)

V is a finite set, called the variables/symbols

Σ is an alphabet / terminals

$S \in V$, called the start variables/symbols

$R \subseteq V \times (V \cup \Sigma)^*$

ex. $R = \{ (S, 0S1), (S, \varepsilon) \}$ $\quad 0S1$

$L(g) = \{ w^{\in \Sigma^*} \mid S \Rightarrow^* w \}$

~~derives x ($V \Rightarrow^* x$) where $V \in V$ and $x \in (V \cup \Sigma)^*$~~

$S \to 0S0 \mid 11X$

$X \to 0X0 \mid 1$

$0^n 1 0^m 1 0^m 0^n$

$S \to 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon$

palindromes

$S_1 \to \ldots$

$S_2 \to \ldots$

$S_3 \to S_1 \mid S_2$

$S_* \to \varepsilon \mid S_1 S_*$

u derives v ($u \Rightarrow^* v$) where $u, v \in (V \cup \Sigma)^*$
iff $u = v$ or $u \Rightarrow x$ and $x \Rightarrow^* v$

u yields x ($u \Rightarrow x$)
iff $u = aAb$ and $x = awb$ and
$(A, w) \in R$ where $a, b, u, w, x \in (V \cup \Sigma)^*$
and $A \in V$

$E \Rightarrow 1 \mid 0 \mid E \times E \mid E + E$

$E \Rightarrow E \times E \Rightarrow 1 \times E$ — 1st E — $a = \varepsilon \quad A = E \quad b = \times E \quad w = 1$

$\Rightarrow E \times 1$ — 2nd E — $a = E\times \quad A = E \quad b = \varepsilon \quad w = 1$

ALT: u yields' x ($u \Rightarrow x$)
iff $u = a'Ab$ and $x = a'wb$ and
$(A, w) \in R$ where $a \in \Sigma^*$ and $b, u, w, x \in (V \cup \Sigma)^*$
and $A \in V$

↳ 2nd option not allowed

↳ "left most derivation"