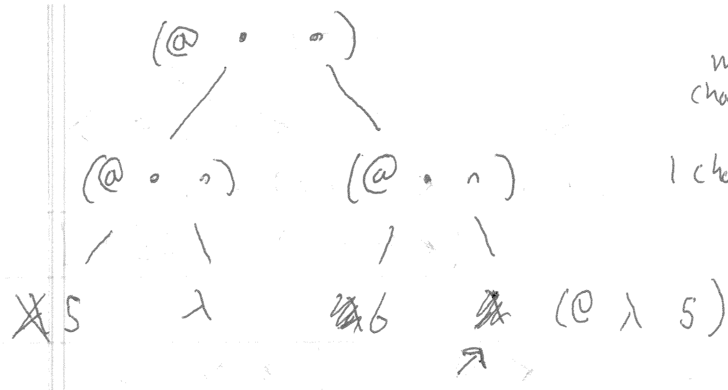
7-1



many choices — original: n steps

1 choice — evaluation context / standard reduction: lg n

$E[M] \longmapsto E[N]$ (M: redex, N: redactum, E: ctxt)

Standard Reduction Algorithm

O(1) 1. Is the term a value? If so, return.

O(n) 2. What is a eval ctxt E s.t. $M = E[(U\ V)]$ or $E[(o^n\ V \ldots)]$

O(n) 3. Performs β_v or δ on the redex $(U\ V)$ or $(o^n\ V \ldots)$, call res N

O(n) 4. Produce $E[N]$

// happens n times 5. Return to step 1.

$M = X$
$\mid \lambda X.M$
$\mid b$
$\mid (o^n\ M \ldots)$
$\mid (M\ M)$

new Var()
new Lam() ✓
new Con() ✓
new Prim() X
new App() X

$M = (+\ (+\ ((\lambda X.X)\ 5)\ ((\lambda X.X)\ 6))\ 7)$

$E = (+\ (+\ \square\ ((\lambda X.X)\ 6))\ 7)$
$U = (\lambda X.X)\quad V = 5$

$E = (+\ (+\ 5\ \square)\ 7)$
$U = (\lambda X.X)\quad V = 6$

$E = (+\ \square\ 7)$
$o^n = +\quad V_1 = 5\quad V_2 = 6$

$E = \square$
$o^n = +\quad V_1 = 11\quad V_2 = 7$

$E = \square$
$\mid (E\ M)$
$\mid (V\ E)$
$\mid (o^n\ V \ldots\ E\ M \ldots)$

PL Semantics to know eval(M) ask $M \longmapsto^* N$

Machine to know eval(M) ask translate(M) $\longmapsto^*$ MachineState ret untran(MS)

$$\begin{tikzcd}
M \arrow[r, "\longmapsto^*_v"] \arrow[d, shift right, "translate"'] & N \arrow[d, shift right, "translate"'] \\
MS \arrow[r, "\longmapsto^*_{ms}"] \arrow[u, shift right, "untrans"'] & MS' \arrow[u, shift right, "untranslate"']
\end{tikzcd}$$

all paths goto same place then correct.

7-2

CC-machine (code string, context)

$St = \langle M, E \rangle$ $\quad$ $\text{translate}(M) = \langle M, \square \rangle$

$\text{untranslate}(\langle M, \square \rangle) = M$

$(\langle M, E \rangle) = E[M]$

1. $\langle (M\ N), E \rangle \longmapsto_{cc} \langle M, E[(\square\ N)] \rangle$ if $M \notin V$
2. $\langle (V\ N), E \rangle \longmapsto_{cc} \langle N, E[(V\ \square)] \rangle$ if $N \notin V$
3. $\langle ((\lambda X.M)\ N), E \rangle \longmapsto_{cc} \langle M[X \leftarrow N], E \rangle$
4. $\langle (o\ V \ldots\ M\ N \ldots), E \rangle \longmapsto_{cc} \langle M, E[(o\ V \ldots\ \square\ N \ldots)] \rangle$ if $M \notin V$

6. $\langle V, E[(U\ \square)] \rangle \longmapsto_{cc} \langle (U\ V), E \rangle$
7. $\langle V, E[(\square\ N)] \rangle \longmapsto_{cc} \langle (V\ N), E \rangle$
8. $\langle V, E[(o^n\ U \ldots\ \square\ N \ldots)] \rangle \longmapsto_{cc} \langle (o^n\ U \ldots\ V\ N \ldots), E \rangle$
5. $\langle (o\ b \ldots), E \rangle \longmapsto_{cc} \langle \delta(o, b \ldots), E \rangle$

1, 2, 4 — structural/parsing $\qquad$ 3, 5 — do work

6, 7, 8 — plugging/unparsing

$((\lambda X.(+\ (+\ 1\ 2)\ X))\ 3) \xrightarrow{tr} \langle ((\lambda X.(+\ (+\ 1\ 2)\ X))\ 3), \square \rangle$

$\xrightarrow{3} \langle (+\ (+\ 1\ 2)\ 3), \square \rangle$

$\xrightarrow{4} \langle (+\ 1\ 2), (+\ \square\ 3) \rangle$

$\xrightarrow{5} \langle 3, (+\ \square\ 3) \rangle \xrightarrow{8} \langle (+\ 3\ 3), \square \rangle \xrightarrow{5} \langle 6, \square \rangle \xrightarrow{un} 6$

S.R. $\quad [124]^*\ [35]\ [678]^*\ [124]^*\ [35]\ \ldots$

C.C. $\quad [124]^*\ [35]\ [124\ 678]^*\ [35]$

CC machine removes plugs that will be immediately parsed again.

$M \longmapsto\!\!\!\!\to_v V$ iff $\langle M, \square \rangle \longmapsto\!\!\!\!\to_{cc} \langle V, \square \rangle$

$\Leftarrow \quad E[M] \longmapsto\!\!\!\!\to_v E[V]$ iff $\langle M, E \rangle \longmapsto\!\!\!\!\to_{cc} \langle V, E \rangle$

7-3

SCC-machine - simplified, code, context

1'. $\langle (M\ N), E \rangle \longmapsto_{scc} \langle M, E[(\square\ N)] \rangle$

4'. $\langle (o\ M\ N \ldots), E \rangle \longmapsto_{scc} \langle M, E[(o\ \square\ N \ldots)] \rangle$

6+3 = 3'. $\langle V, E[(\lambda X.M)\ \square] \rangle \longmapsto_{scc} \langle M[X \leftarrow V], E \rangle$

2+7 = 2'. $\langle V, E[\square\ N] \rangle \longmapsto_{scc} \langle N, E[(V\ \square)] \rangle$

8+5 = 5'. $\langle b_n, E[(o^n\ b_1 \ldots b_{n-1})] \rangle \longmapsto_{scc} \langle \delta(o^n, b_1 \ldots b_n), E \rangle$

3+4 = 8'. $\langle V, E[(o^n\ U \ldots \square\ M\ N \ldots)] \rangle \longmapsto_{scc} \langle M, E[(o^n\ U \ldots V\ \square\ N \ldots)] \rangle$

SCC strictly shorter than the CC because it fuses steps
no longer uses $\in V$?

Still expensive! but

2'. $\langle V, E[(\square\ N)] \rangle \longmapsto_{scc} \langle N, E[(V\ \square)] \rangle$ easy to fix.

$V = 6 \quad N = 8$

$E = (+ (+ (+ (+ (+ (+ (+\ \square \ldots) \ldots) \ldots) \ldots) \ldots)$

$E[(\square\ N)] = (+ (+ + ((+ (+ (+ (+\ \underbrace{(\square\ N)} \ldots) \ldots) \ldots) \ldots) \ldots)$

$E[(V\ \square)] = (+ (+ (+ (+ (+ (+ (+ (V\ \square) \ldots) \ldots) \ldots) \ldots) \ldots)$

Small-step: all the examples so far $(\rightarrow, \twoheadrightarrow, \longmapsto, \longmapsto\!\!\!\!\!\!\twoheadrightarrow)$

Big-step semantics: like $\twoheadrightarrow$, but restricted to the RHS being a value. $(\Downarrow)$

$$\frac{}{\lambda X.M \Downarrow \lambda X.M} \qquad \frac{}{b \Downarrow b} \qquad \frac{M \Downarrow \lambda X.O \quad N \Downarrow V}{(M\ N) \Downarrow O[X \leftarrow V]}$$

$$\frac{M_1 \Downarrow b_1 \quad \ldots \quad M_n \Downarrow b_n}{(o^n\ M_1 \ldots M_n) \Downarrow \delta(o^n, b_1 \ldots b_n)}$$

$\Downarrow$ = standard interpreter