6-1

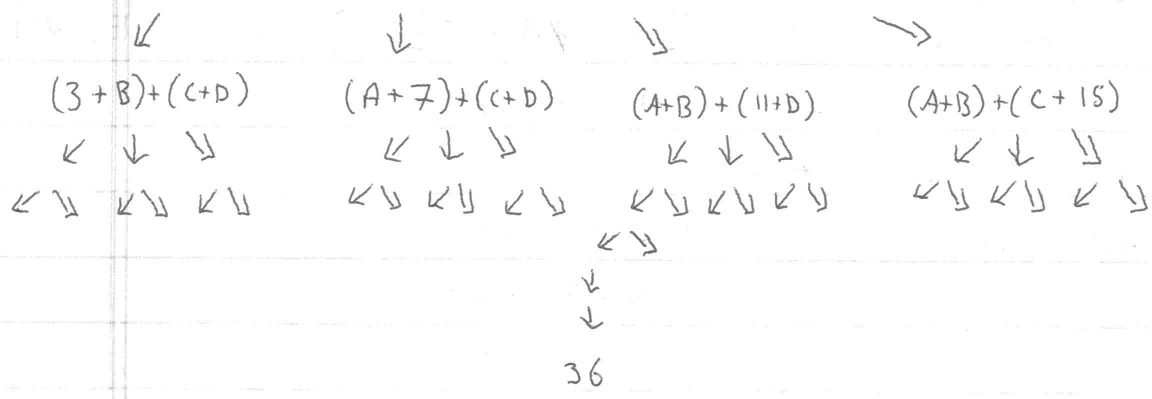
$M = X \mid \lambda X. M \mid (M\ N) \mid b \mid (o^n\ M \ldots)$

$V = \lambda X. M \mid b$

$C = [\,] \mid \lambda X. C \mid (C\ N) \mid (M\ C) \mid (o^n\ M \ldots C\ M \ldots)$

$\beta_v : \ (\lambda X. M)\ V \longrightarrow M[X \leftarrow V]$

$$\overset{A}{((1+2)} + \overset{B}{(3+4))} + (\overset{C}{(5+6)} + \overset{D}{(7+8)}))$$



interp* : $M \to M$

interp* m = m if interp m is false
o.w. interp* (interp m)

n^3 where n is size of program

interp : $M \Rightarrow M$ or #false

interp m = OR (if m is $((\lambda X.M)\ V)$ then $m[X \leftarrow V]$)
(if m is $(\lambda X. M)$ and m' = interp M, then $\lambda X. m'$)
(if m is $(M\ N)$ and m' = interp M, then $(m'\ N)$)
(if m is $(M\ N)$ and m' = interp N, then $(M\ m')$)
(if m is $(o^n\ N \ldots\ M\ O \ldots)$ and m' = interp M, then
$(o^n\ N \ldots\ m'\ O \ldots)$)

Evaluation Context – the place where work happens

$E ::= \square \mid \quad \mid (E\ N) \mid (V\ E) \mid o^n\ V \ldots E M \ldots$

$C ::= \square \mid (\lambda X. C) \mid (C\ N) \mid (M\ C) \mid o^n\ M \ldots C M \ldots$

$\longmapsto_v$ (the standard reduction)

$M \longmapsto_v N$ iff $\exists E.\ M = E[M']$ and $N = E[N']$ and $M'\ v\ N'$

interp : M ⇒ M or #false

interp (M N) = if $M \in V$ then
 if $N \in V$ then
 beta M N
 o.w. (M (interp N))
 o.w. ((interp M) N)

interp $(o^n\ V \ldots\ M_0\ M \ldots)$ = $(o^n\ V \ldots\ (\text{interp } M_0)\ M \ldots)$

Uniqueness of Eval Ctxts.

$\forall M.\ M = V$ or there exists a unique E s.t.
$M = E[(V_1\ V_2)]$ or $M = E[(o^n\ V_1 \ldots V_n)]$

Correctness

(old way) (new way)

$\forall M$ and U. $M \longrightarrow\!\!\!\!\rightarrow_v U$ iff $M \longmapsto\!\!\!\!\rightarrow_v V$ and $V \longrightarrow\!\!\!\!\rightarrow_v U$

Stuck: M is stuck if
$M = (o^n\ b_1 \ldots b_n)$ and $\delta(o^n, b_1, \ldots, b_n)$ is undefined
or $M = (o^n\ b_1 \ldots (\lambda X. M)\ V \ldots)$
or $M = (b\ V)$

Uniform Evaluation Theorem $M \longmapsto\!\!\!\!\rightarrow_v V$

If M is closed ($FV(M) = \emptyset$), then $M \longmapsto\!\!\!\!\rightarrow_v N$ where N is stuck
$(\forall N.\ M \longmapsto\!\!\!\!\rightarrow_v N \Rightarrow \exists L.\ N \longmapsto_v L)$