23-1

ISWIM identity function

$= \lambda x.x$

$(\lambda x.x)\ 5 \longrightarrow^* 5$

$((\lambda x.x)\ (\lambda y.\ y+5))\ 0 \longrightarrow^* 5$

Typed-ISWIM family of identity funs

$(\lambda x:num.\ x)\ 5 \to^* 5$

$((\lambda x:(num \to num)\ x)\ (\lambda y:num.\ y+5))\ 0 \to^* 5$

Polymorphism "many shapes" a term with many types
a thing with many shapes

$M := \ldots \mid (\Lambda A.\ M) \mid M[T]$ $A :=$ some set distinct from X

$V := \ldots \mid (\Lambda A.\ m)$

$T := \ldots \mid A \mid \forall A.\ T$ $E := \ldots \mid E[T]$

β_T. $(\Lambda A.\ m)[T] \longrightarrow m[A \leftarrow T]$

$id := \Lambda A.\ (\lambda x:A.\ x)$ $(id[num]\ 5) \to^* 5$

$(id[num \to num]\ (\lambda y:num.\ y+5))\ 0 \to^* 5$

$(id[num]\ 5) \to ((\lambda x:num.\ x)\ 5) \longrightarrow 5$

$((\lambda id:(\forall A.A \to A).\ (id[num]\ 5) + ((id[num \to num]\ (\lambda y:num.\ y+5))\ 0))$
$(\Lambda A.\ (\lambda x:A.\ x)))$ $\longrightarrow^* 10$

3-2/ $\Gamma ::= \cdot \mid \Gamma, X:T \mid \Gamma, A$ $\quad (\Lambda A.\ 5) : \forall A.\ num$

$(\lambda x:B.\ x)$

$$\frac{\Gamma, A \vdash M : T}{\Gamma \vdash (\Lambda A.\ M) : \forall A.\ T} \qquad \frac{\Gamma \vdash M : \forall A.\ T' \quad \Gamma \vdash T}{\Gamma \vdash M[T] : T'[A \leftarrow T]}$$

$$\Gamma \vdash num \qquad \Gamma \vdash bool \qquad \frac{\Gamma \vdash T_1 \quad \Gamma \vdash T_2}{\Gamma \vdash T_1 \to T_2} \qquad \frac{A \in \Gamma}{\Gamma \vdash A} \qquad \frac{\Gamma, A \vdash T}{\Gamma \vdash \forall A.\ T}$$

$$\frac{\Gamma, X:T_1 \vdash M : T_2 \quad \Gamma \vdash T_1}{\Gamma \vdash (\lambda X:T_1.\ M) : T_1 \to T_2}$$

OLD: $\dfrac{\Gamma \vdash M : T_1 \to T_2 \quad \Gamma \vdash N : T_1}{\Gamma \vdash (M\ N) : T_2}$

$M = (\lambda f : (\forall A.\ A \to A).\ (f[num]\ 5))$

$N = (\Lambda B.\ (\lambda x:B.\ x)) : \forall B.\ B \to B$

EW: $$\frac{\Gamma \vdash M : T_1^\circ \to T_2 \quad \Gamma \vdash N : T_1' \quad \vdash T_1^\circ \leftrightarrow T_1'}{\Gamma \vdash (M\ N) : T_2}$$

$$\vdash T \leftrightarrow T \qquad \frac{\vdash T_1^\circ \leftrightarrow T_1' \quad \vdash T_2^\circ \leftrightarrow T_2'}{\vdash (T_1^\circ \to T_2^\circ) \leftrightarrow (T_1' \leftrightarrow T_2')}$$

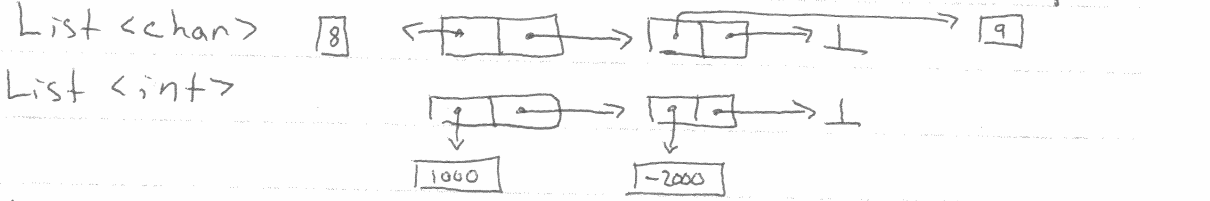
$$\frac{\vdash T^\circ[A^\circ \leftarrow B] \leftrightarrow T'[A' \leftarrow B]}{\vdash (\forall A^\circ.\ T^\circ) \leftrightarrow (\forall A'.\ T')} \qquad B \notin FV(T^\circ) \cup FV(T')$$

Polymorphism in Languages, Real:

Java — Generics List<X>
C++ — Templates
Haskell / ML — matches theory

Compiler C C(ΛA.m) = ?

Strategy 1 (Java):
- compile M one-time with a uniform value representation



- Advantage: one function at runtime and one-compiled
- Dis: use more memory, take more time, messes with cache

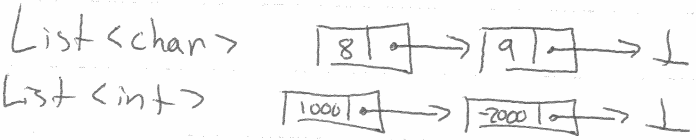
Strategy 2 (C++):
- statically copy the code

ΛA.m can only appear in (λx:T. m) or in m[T]
these spots
(m N)

((snd (pair 5 (ΛA. λx:A.x))) [num] 8) →* 8

N, m := x | b | or m ... | m N | λx:T. m | m̂ [T]
N̂, m̂ := x | ΛA. m | ~~(λx:T. m)~~ N̂ | λx:T. m̂ | m̂ [T]

C((λx:T. m) m̂) → C(m[x←m̂])
C((ΛA.m) [T]) → C(m[A←T])



Adv: less memory, less time
Dis: compilation longer
code binary bigger
same rep, diff impl

3-4 / Parametricity

$(\lambda x{:}num.\ x+1) : num \to num$

$(\lambda x{:}num.\ x) : num \to num$

mostly →

$f : \forall A.\ A \to A.$

$(f\ [num]\ \ 5) \to^* 5$

$f = \Lambda A.m$ $\qquad m = \lambda X{:}A.\ N$ $\qquad N = X$

$$\frac{\Gamma \vdash (\Lambda A.\ m) : \forall A.\ A \to A}{\Gamma, A \vdash m : A \to A} \qquad \frac{\Gamma, A, X{:}A \vdash N : A}{\Gamma, A \vdash \lambda X{:}A.\ N : A \to A}$$

$map : \forall A.\ \forall B.\ (A \to B) \to (List\ A) \to (List\ B)$