1-1

Theory of Programming Languages

Is it O-O? Functional? $\underbrace{}_{\text{paradigm}}$

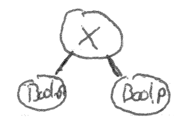
Interpreted? Compiled? $\underbrace{}_{\text{implementation detail}}$

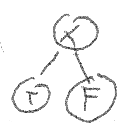
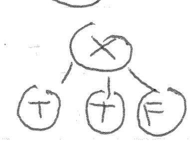
What do function calls look like? $\underbrace{}_{\text{syntax}}$

<u>Semantics</u> — a mathematical definition of meaning
a generalization of "1 plus 1 is 2"
a ~~function~~ relation of $\underbrace{\text{syntax}}_{\text{programs}}$ and answers

Arithmetic is a language ("1 plus 1", 2) ∈ Arith

What is a PL? — programs, answers, the semantics

Bool. P = programs of Bool = an infinite set of trees
= T or F or 

H ∉ B.P T ∈ B.P  ∈ B.P  ∉ B.P

CFG
= true | false | (x B.P B.P)

h ∉ B.P true ∈ B.P (x true false) ∈ B.P (x t ~~f~~ f) ∉ B.P
= T | F | B.P x B.P

Bool. A = answers (meanings) = another set of trees
= T | F $\boxed{\forall \alpha, (F \times \alpha) \text{ means } F}$

Semantics is a relation between P and A
- interpreter — a program in X which maps P to A
- operational — an interpreter where X = Math
- denotational — a compiler from P to Math

We do operational semantics

r is our semantics relation

$P \; r \; A$ is a semantics judgement

$(P, A) \in r$ $\quad\quad\quad\quad$ $r \subseteq P \times A$

r1. $\forall B. \quad (f \times B) \; r \; f$

r2. $\forall B. \quad (t \times B) \; r \; B$

r1 $\quad (f \times f, f) \in r \quad (f \times (f \times t), f) \in r$

reflexive closure of r is $r \cup (\forall x. \; (x, x))$

$\quad$ refl $r \subseteq P \times P$

symmetric closure: $a \; r \; b \rightarrow b \; r \; a$

transitive closure: $a \; r \; b \wedge b \; r \; c \Rightarrow a \; r \; c$

rst $(r1 \cup r2) = r^*$

$((t \times (f \times t)), f) \in r^*$

r2 $\ni ((t \times (f \times t)), (f \times t))$

r1 $\ni ((f \times t), f)$

<u>compatible closure</u>

L $\quad a \; r \; b \quad \Rightarrow (B \times a) \; r \; (B \times b)$

R $\quad a \; r \; b \quad \rightarrow (a \times B) \; r \; (b \times B)$

$(t \times \overbrace{(f \times (t \times \underbrace{(f \times t)}_{\downarrow \, f}))}^{f})$

eval: evaluation function $P \rightarrow A$

eval$(P) = A$ iff $(P, A) \in$ rst$(cc(r1 \cup r2))$

$P \rightarrow^* A$