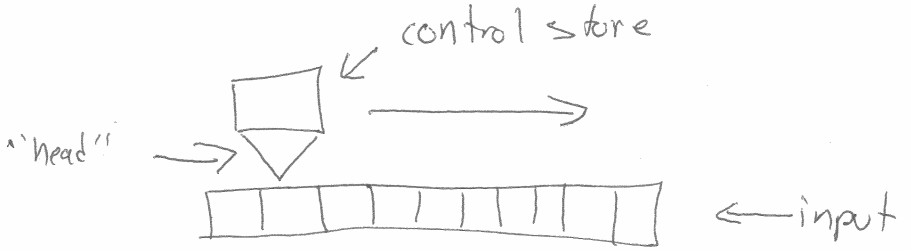
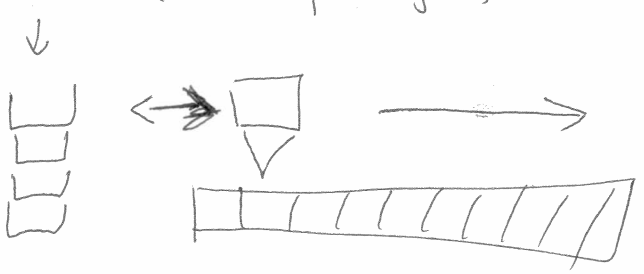
DFA : REX :: PDA : CFG

Push-down Automata

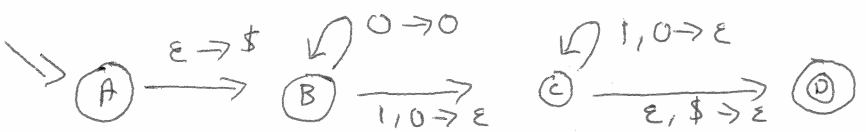
DFA



PDA



$0^n 1^n$ $\quad S \to \varepsilon \mid 0\,S\,1$

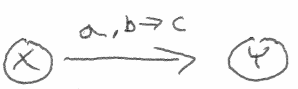


$a \to c = a, \varepsilon \to c$

DFA

In state X, on an 'a' in input, goto Y

PDA



In state X, on an 'a' in input with 'b' on stack top, goto Y and replace stack top with 'c'

in: 000111 stack [state] input remaining

[A]000111 => $[B]000111 => $0[B]00111 => =>
$000[B]111 => $00[C]11 => => $[C] => [D]

[A]11 => $[B]11

PDA $P = (Q, \Sigma, \Gamma, q_0, \delta, F)$

Q = finite states

Σ = alphabet (input)

Γ = stack alphabet = $\{0, \$\}$

$q_0 \in Q$

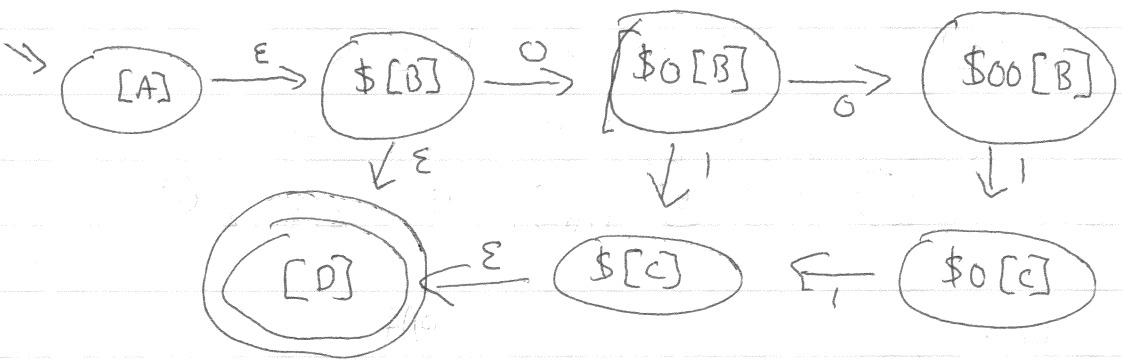
$F \subseteq Q$

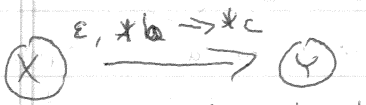
$\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \rightarrow P(Q \times \Gamma_\varepsilon)$

$\delta(A, 0, \varepsilon) = \{(B, 0)\}$

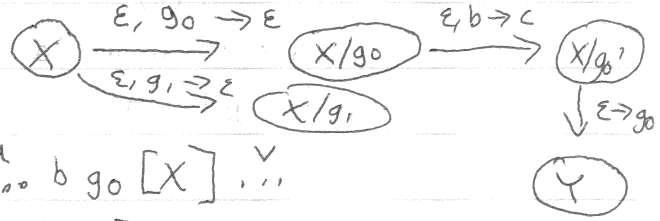
In A, on 0, ignore stack, goto B, push 0

	read input	ignore input
push :	$a \rightarrow c$	$\varepsilon \rightarrow c$
pop :	$a, b \rightarrow \varepsilon$	$\varepsilon, b \rightarrow \varepsilon$
replace :	$a, b \rightarrow c$	$\varepsilon, b \rightarrow c$





On X, if 2nd stack is b, replace with c, and goto Y



u ..... b g0 [X] v

u b g0 [X] v

⇒ u b [X/g0] v

⇒ u c [X/g0'] v

⇒ u c g0 [Y] v

9-3/

$$L(P) = \{ w \mid \varepsilon[q_0]w \Rightarrow^* u[q_f]\varepsilon \text{ s.t. } q_f \in F \}$$

$$ub[q_i]av \Rightarrow uc[q_j]v \quad \text{iff} \quad (q_j, c) \in \delta(q_i, a, b)$$
$$a \in \Sigma_\varepsilon \quad b, c \in \Gamma_\varepsilon$$

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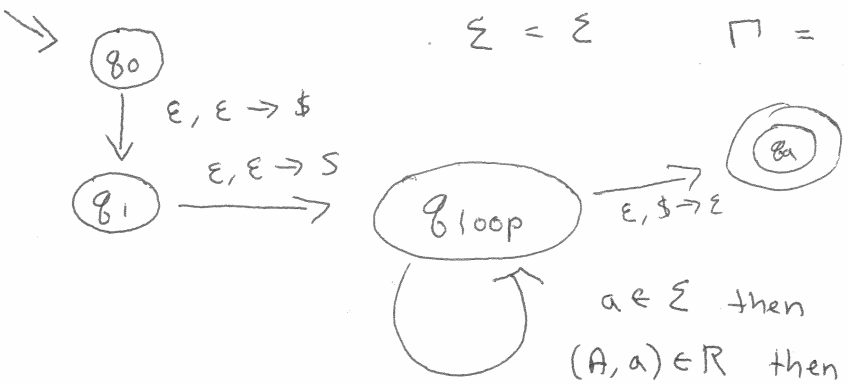
① CFG $\Rightarrow$ PDA $= (V, \Sigma, R, S)$ ② PDA $\Rightarrow$ CFG $= (Q, \Sigma, \Gamma, q_0, \delta, F)$

① Assume that the CFG is in CNF

If "$S \Rightarrow \varepsilon$" $\in R$, then $q_0 \in F$

$\Sigma = \Sigma \quad \Gamma = V \cup \Sigma \cup \{\$\}$



$a \in \Sigma$ then $a, a \Rightarrow \varepsilon$

$(A, a) \in R$ then $\varepsilon, A \Rightarrow a$

$(A, BC) \in R$ then $\varepsilon, A \Rightarrow CB$ (top of stack)

$[q_0]000111 \Rightarrow \$S'[q_{loop}]000111 \Rightarrow \$R[q_e]000111$

~~$\$XS[q_e]000111$~~

$\Rightarrow \$R0[q_e]000111$

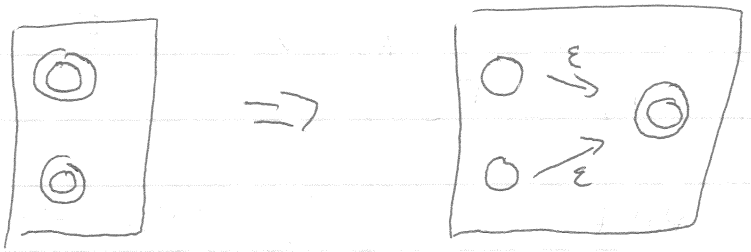
$\$YRX[q_e]00111 \Leftarrow \$YS[q_e]00111 \Leftarrow \$R[q_e]00111$

$\Rightarrow \$YR0[q_e]00111 \Rightarrow \$YR[q_e]0111$

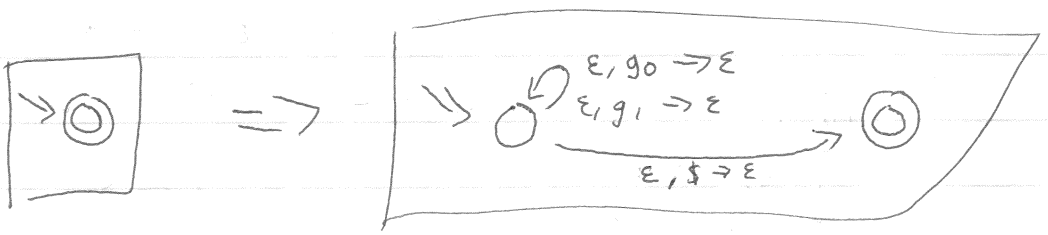
9-4/

Turn a PDA $p = (Q, \Sigma, \Gamma, q_0, \delta, F)$
into a CFG $g = (V, \Sigma, R, S)$

Assume that p has one accept state ($F = \{q_f\}$)



Assume that p empties the stack before accepting



Assume that every transition pushes OR pops

X | a, ε → ε | Y
a, b → ε | [pop]
a, ε → c | [push]

X | a, b → c | Y

$$V = \{ A_{p,q} \mid p, q \in Q \}$$

$A_{p,q} \Rightarrow^* w$ if $[p]\, w \Rightarrow^* [q]$

To be continued...