8-1

$\forall n \in \mathbb{N}.\ 0^n 1^n \in A$ $\notin REG$

$\forall x \in \mathbb{N}.\ (0^n 1^n \ulcorner x \urcorner)$ $\ulcorner x \urcorner = x$ is binary

$\{w \in \{0,1\}^* \mid w$ has equal nums of 0s and 1s$\} \notin REG$

$\{ww \mid w \in \Sigma^*\}$
$\{ww^R \mid w \in \Sigma^*\}$ $\notin REG$

$\{0^{n^2} \mid n \in \mathbb{N}\}$
$\{0^i 1^j \mid i \leq j\}$

$\{0^x 1 0^y 1 0^{x+y}\} \notin REG$

Context-free Grammar "$:$" Regular Expression → lang : set(strings) $::$

"$:$" DFA → accepter : string → bool

$0^n 1^n$

$S \to \varepsilon \mid 0S1$

$0^x 1 0^y 1 0^{x+y}$

$S \to 1R_{(A)} \mid 0S0_{(B)}$
$R \to 1_{(C)} \mid 0R0_{(D)}$

$S \xrightarrow{B} 0S0 \xrightarrow{B} 00S00 \xrightarrow{A} 001R00$
$\xrightarrow{D} 0010R000 \xrightarrow{D} 00100R0000$
$\xrightarrow{D} 001000R00000 \xrightarrow{C} 0010001000000$

~~[illegible]~~ palindromes

$S \to 0 \mid 1 \mid \varepsilon \mid$
$0S0 \mid 1S1$

"Racket"

$S \to \varepsilon \mid (S) \mid SS$

$0^n 1^n$ CNF

1) $S' \to S$ $\quad S \to \varepsilon \mid 0S1$
2) $S' \to S \mid \varepsilon$ $\quad S \to 01 \mid 0S1$
3) $S' \to 01 \mid 0S1 \mid \varepsilon$ $\quad S \to 01 \mid 0S1$
4) $S' \to XY \mid XR \mid \varepsilon$ $\quad X \to 0$ $\quad Y \to 1$
$S \to XY \mid XR$ $\quad R \to SY$

1-2

CFG has ...

- variables/symbols = S, R
- rules
  substitution = $S \Rightarrow 1R$
  derivation
  $\underbrace{S}_{\text{var}} \Rightarrow \underbrace{1R}_{\text{string of vars and terms}}$
- terminals = 0, 1
- start variable = S

$S \Rightarrow 1R \mid 0S0$
$R \Rightarrow 1 \mid 0R0$

$S \Rightarrow 1R$
$S \Rightarrow 0S0$
$R \Rightarrow 1$
$R \Rightarrow 0R0$

$S \Rightarrow 0 \mid 0S$

$S \Rightarrow 0 \mid R$
$R \Rightarrow 0R$
$S \Rightarrow 0S$

CFG $g = (V, \Sigma, R, S)$

V = some finite set

Σ = alphabet

$S \in V$

$R \subseteq (V \times (V \cup \Sigma)^*)$ (a relation on V and strings of V and Σ)

$L(g) = \{ w \in \Sigma^* \underbrace{S \Rightarrow^* w}_{S \text{ derives } w} \}$

u derives v ($u \Rightarrow^* v$) iff
  $u = v$ or $u \Rightarrow x$ and $x \Rightarrow^* v$
  ⤷ yields

uAv yields uwv ($uAv \Rightarrow uwv$) iff
  $(A, w) \in R$

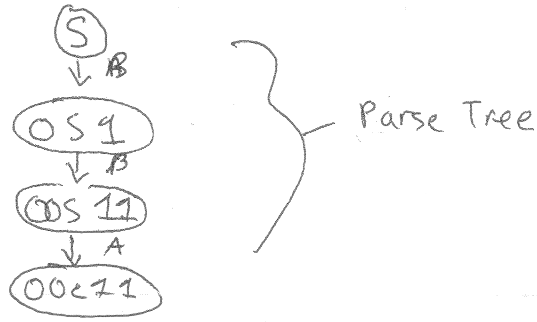
$w, v, u \in (\Sigma \cup V)^*$
$A \in V$

"0SS1"
1) $u = 0S$ $A = S$ $v = 1$
2) $u = 0$ $A = S$ $v = S1$

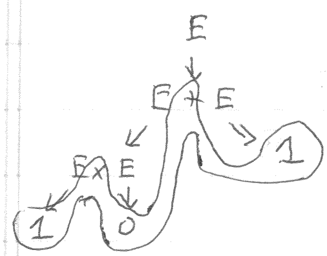
left-most-derivation
  $u \in \Sigma^*$

8-3

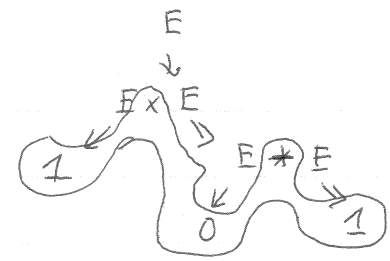
$S \to \underset{A}{\varepsilon} \mid \underset{B}{0S1}$ $(0^n 1^n)$



$E \to 0 \mid 1 \mid E+E \mid E \times E$



$= 1 \times 0 + 1 =$



$E \to 0 \mid 1 \mid (E+E) \mid (E \times E)$

$((1 \times 0) + 1)$ = L

$(1 \times (0 + 1))$ = R

Attribute Grammar (bottom-up) = CFG + output

$E \to 0$ [output $= 0_8$]

$E \to 1$ [out $= 1_8$]

$E \to E+E$ [out = \$0 $+_8$ \$1]

$E \to E \times E$ [out = \$0 $\times_8$ \$1]

If a CFG g has different parse trees for $w \in \Sigma^*$, then w is ambiguous and g is ambiguous

Some gs can be unambiguous, but not all.

($\exists$ a g where $\forall g'.\ L(g') = L(g)$, g' is ambiguous)

8-4

DFA ⊂ CFG?

i.e. can we translate a DFA into a CFG?

REX =	$c \in \Sigma$	$\longrightarrow S \Rightarrow c$
$\mid$	$\emptyset$	$\longrightarrow \sqcup$ OR $S \Rightarrow S$
$\mid$	ε	$\longrightarrow S \Rightarrow \varepsilon$
$\mid$	$r_x \circ r_y$	$\longrightarrow S \Rightarrow S_{\circ x} \; S_{\circ y}$
$\mid$	$r_x \cup r_y$	$\longrightarrow S \Rightarrow S_{\circ x} \mid S_{\circ y}$
$\mid$	r_x^*	$\longrightarrow S \Rightarrow \varepsilon \mid S_{\circ x} \; S$

$(0 \cup 1) \circ 1^* \longrightarrow S \rightarrow XY$

$X \rightarrow A \mid B$

$A \rightarrow 0$

$B \rightarrow 1$

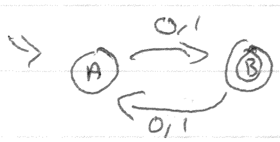
$Y \rightarrow \varepsilon \mid CY$

$C \rightarrow 1$

$S \Rightarrow XY$

$X \Rightarrow 0 \mid 1$

$Y \Rightarrow \varepsilon \mid 1Y$



$A \rightarrow 0B \mid 1B$

$B \rightarrow \varepsilon \mid 0A \mid 1A$

$S \Rightarrow 0S1S001 \mid \varepsilon$

Chomsky-Normal Form is a special case of CFGs

Every rule is either:

$S \Rightarrow \varepsilon$

$A \Rightarrow a \qquad a \in \Sigma$

$A \Rightarrow BC \qquad B, C \in V$

B and $C \neq S$

$\boxed{\forall g, \exists g', g' \in CNF \text{ AND } L(g') = L(g)}$

1. Add a new start state $(S' \Rightarrow S)$
2. Remove ε-rules $(A \Rightarrow \varepsilon)\ (B \Rightarrow AC) \Rightarrow (B \Rightarrow AC)\ (B \Rightarrow C)$
3. Remove unit rules $(A \Rightarrow D)\ (B \Rightarrow AC) \Rightarrow (B \Rightarrow DC)\ (B \Rightarrow AC)$
4. Add "intermediate" symbols $(A \Rightarrow BCD) \Rightarrow (A \Rightarrow XD)\ (X \Rightarrow BC)$