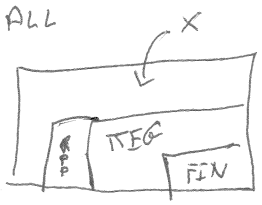
7-1



"P" = $\varepsilon \in x$

$\exists x \in ALL.\ x \notin REG$,

$\forall d \in DFA.\ L(d) \neq x$ "P"$_2$ = $|x| \in$ infinite ordinal i.e. $\aleph_i$ for some i

lemma: $\exists P.\ \forall x \in REG.\ P(x)$

burden: $\exists x \in ALL.\ \neg P(x)$

$\Rightarrow$ $x \notin REG$ $\exists P.\ \forall d \in DFA.\ P(L(d))$ $P: LANG \to PROP$

$\exists P'.\ \forall d \in DFA.\ P'(d)$ $P': DFA \to PROP$

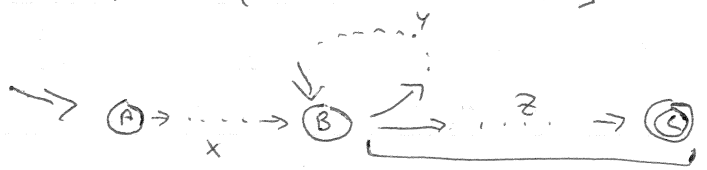
Suppose d accepts string w (i.e. $w \in L(d)$)

How many states could d visit during its run?

$[1, |w|+1]$

Suppose d had 4 states and $w = 01200$

$[1, \min(|Q|, |w|+1)] = [1, 4]$



$x, y, z \in \Sigma^*$

$|x|$ could be 0

$|z|$ could be 0

if $w \in L(d)$

$A = B$ $B = C$

① $w = xyz$

If $w \in L(d)$ exist, ② $|xy| < |Q|$ ③ $|y| > 0$ $= P'$

and $|w| > |Q|$ $\Rightarrow$

④ $xz \in L(d)$, $xyyz \in L(d)$

$\forall i \in N.\ xy^i z \in L(d)$

$P = \exists_{p \in N}.\ P'$ (replace $|Q|$ with p)

Regular Pumping Property (RPP)

: (A : language over Σ) → Prop :=

$\exists p \in \mathbb{N}.$
$\forall (w \in A. \mid |w| \geq p)$
$\exists (x, y, z \in \Sigma^* \mid w = xyz \wedge |xy| \leq p \wedge |y| > 0).$
$\forall i \in \mathbb{N}.$
$xy^i z \in A.$

$(\forall A \in REG.\ RPP(A))$ ← previous page ($p = |Q|$, xyz were based on DFA)

$\neg RPP(A) :=$
$\forall p \in \mathbb{N}.$
$\exists (w \in A \mid |w| \geq p)$
$\forall (xyz \in \Sigma^* \mid w = xyz \wedge |xy| \leq p \wedge |y| > 0)$
$\exists i \in \mathbb{N}.$
$xy^i z \notin A.$

$A = 10^*01$ · given : p · choose : $w = 10^p01$

given : x, y, z $w = xyz$ $|xy| \leq p$ $|y| > 0$

$\therefore$ $xy = 10^a$ $z = 0^b01$ $a + b = p$

$1 + a \leq p$ $x = 10^c$ $y = 0^d$ $a = c + d$

$d > 0$

$xy^i z = 10^c 0^{di} 0^b 01$ $c + di + b \in \mathbb{N}$

7-3

((( )) ␣ ∈ RACKET

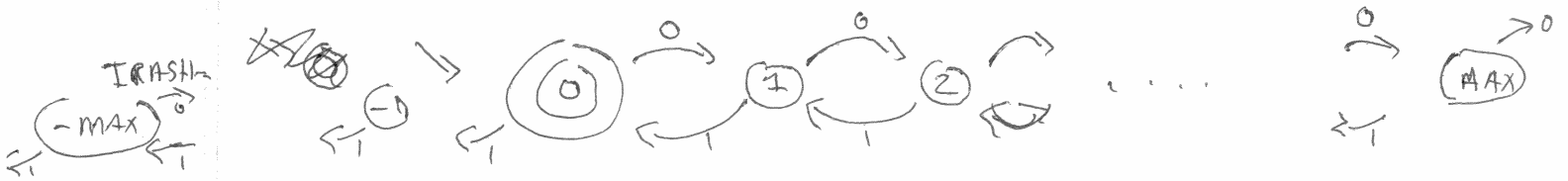
(( ) ( ) (( ))) ∈ RACKET = more complicated

( )) (∈ TRASH = more complicated

(( )) ∈ BABY = $(^n)^n$

0 0 1 1 ∈ ASM = $0^n 1^n$

```
int sum=0
while ((char c = getc()) ≠ EOF)                    = TRASH
 if (c) { sum++; } else { sum--; } }
return (sum == 0);
```



ASM ∉ REG.

Given : p Choose : $w = 0^p 1^p$ $|w| = 2p$

Given : x y z , $w = xyz$, $|xy| \leq p$ $|y| > 0$

$x = 0^a$ $y = 0^b$ $z = 0^c 1^p$

$a+b \leq p$ $a+b+c = p$ $b > 0$

$xy^i z = 0^a 0^{bi} 0^c 1^p = 0^{a+bi+c} 1^p$

$\in$ ASM if $a+bi+c = p$

$bi - b = 0$ $b(i-1) = 0$

$i = 1$

choose : $i \neq 1$

$\Rightarrow$ ASM ∉ REG

and REG ≠ ALL