D: NFA → REX 6/1

input: n which is a k-state NFA

$1 \xrightarrow{\emptyset} 2$

$D(n) = IN \cdot RIP^k \cdot OUT$

IN: k-state NFA → k+2 state GNFA

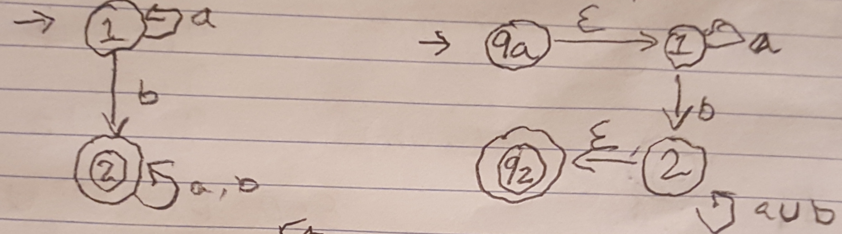
RIP: k+1 state GNFA → k-state GNFA

OUT: 2-state GNFA → REX

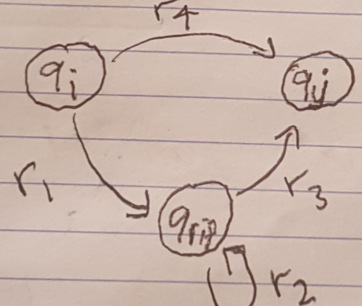
IN = add states q_a and q_z

connect q_a to q_0 with ε

connect $q_f \in F$ to q_z with ε



RIP =

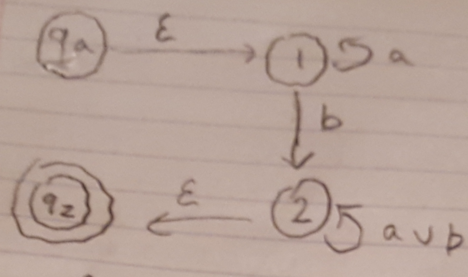


$q_i \xrightarrow{r_1 r_2^* r_3 \cup r_4} q_j$

$\emptyset \cup X = X$

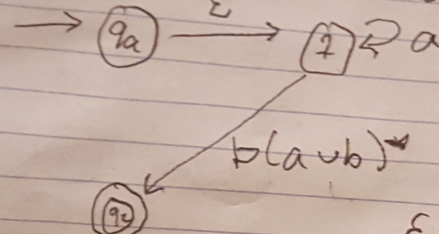
$\emptyset^* = \varepsilon$

$X\emptyset = \emptyset = \emptyset X$



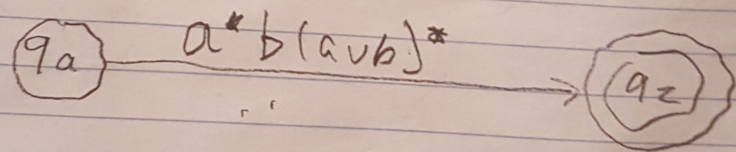
$q_{rip} = 2$
$q_i = 1$
$q_j = q_z$

$\emptyset \cup \varepsilon$
$\emptyset \cup \emptyset$
$b \cdot (a \cup b)^* \varepsilon \cup \emptyset$



$q_{rip} = 1$
$q_i = q_a$
$q_j = q_z$

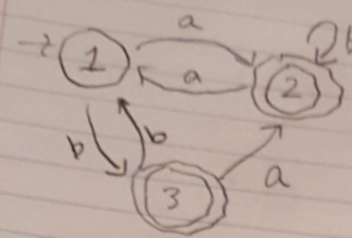
$\varepsilon \cdot \underbrace{a^* b(a \cup b)^*} \cup \emptyset$

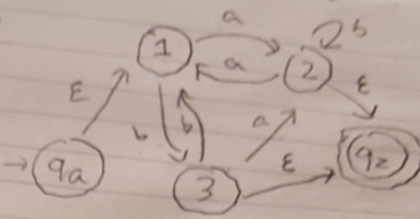


OUT: 2-state GNFA → REX
$\Delta(q_a, q_z) = $ REX

return $a^* b (a \cup b)^*$

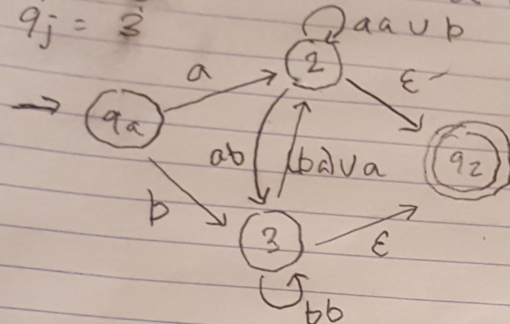
6/3

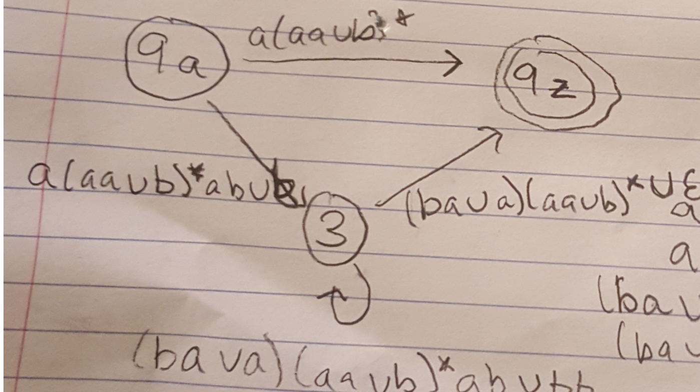




$q_{rip} = 1$
$q_i = 3$
$q_j = 3$

$\varepsilon \cdot \varepsilon \cdot a \cup \emptyset$
$a \cdot \varepsilon \cdot a \cup b$
$b \cdot \varepsilon \cdot a \cup a$
$a \cdot \varepsilon \cdot b \cup \emptyset$
$\varepsilon \cdot \varepsilon \cdot b \cup \emptyset$
$b \cdot \varepsilon \cdot b \cup \emptyset$

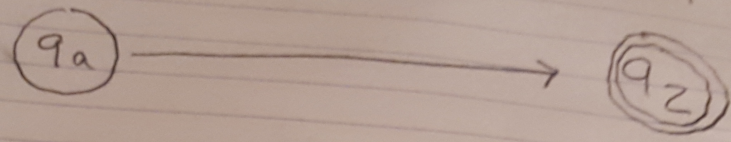




$q_{rip} = 2$
$q_i = 3$
$q_j = q_z$

$a(aa \cup b)^* \varepsilon \cup \emptyset$
$a(aa \cup b)^* ab \cup b$
$(ba \cup a) \cdot (aa \cup b)^* ab \cup bb$
$(ba \cup a) \cdot (aa \cup b)^* \varepsilon \cup \varepsilon$

6/9

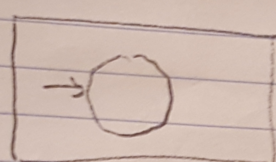


$q_{rip} = 3 \quad q_i = q_a \quad q_j = q_z$

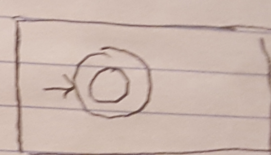
$(a(aa \cup b)^* ab \cup b) \cdot ((ba \cup a)(aa \cup b)^* ab \cup bb)^* \cdot ((ba \cup a)(aa \cup b)^* \cup \varepsilon) \cup a(aa \cup b)^*$

REX → NFA

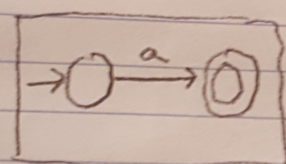
$r = \emptyset$



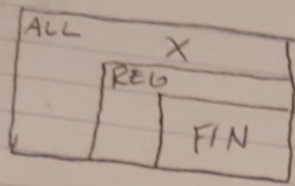
$r = \varepsilon$



$r = a \in \Sigma$



6/5



$X \in ALL$
$X \notin REG$

Prove: $\exists x.\ X \in ALL$ but $\underbrace{X \notin REG}_{\neg(X \in REG)}$

$\neg(A \wedge B) = \neg A \vee \neg B$

$\neg(\exists x.\ P(x)) = \forall x.\ \neg P(x)$

$\neg(\forall x.\ P(x)) = \exists x.\ \neg P(x)$

$\neg(\exists d \in DFA.\ L(d) = x)$

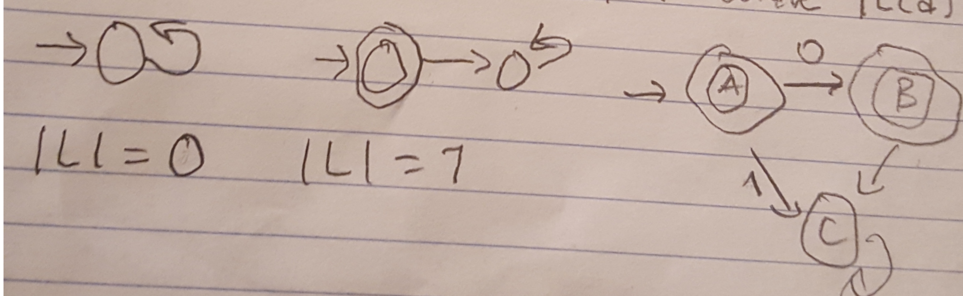
$\forall d \in DFA.\ L(d) \neq X$

$\exists x \in P(\Sigma^*).\ \forall d \in DFA.\ L(d) \neq x$

Imagine $F: DFA \to Prop$ and $\forall d \in DFA\ F(d) =$ true

Suppose $F': Lang \to Prop$
and $\forall d \in DFA.\ F'(L(d)) =$ true
Suppose that $\neg F'(x)$
implies that $x \notin DFA$

What is the smallest DFA where $|L(d)| = 2$



$|L| = 0$ $\quad$ $|L| = 1$

$\varepsilon ; 0$

Suppose d has many states, and $x \in L(d)$

How many states could x visit?

~~$[1, |x|]$~~

$[1, 1+|x|]$