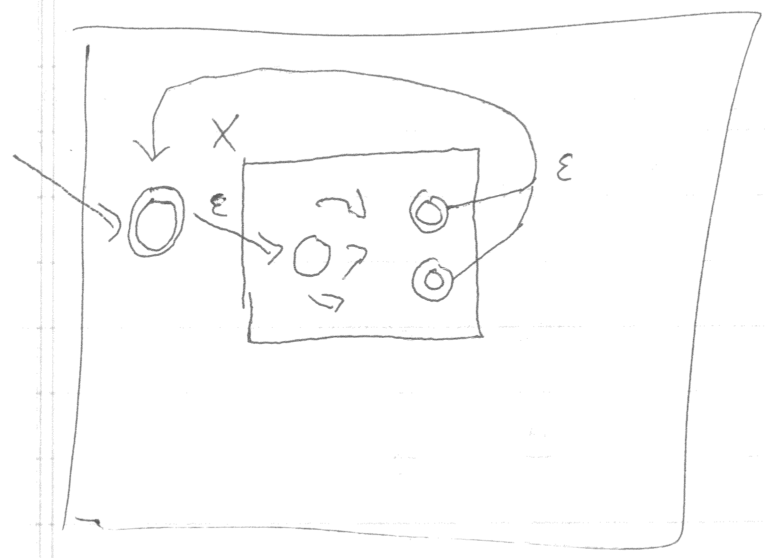
X^*

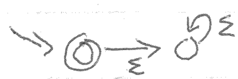


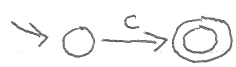
$X \cup Y$, X^*, $X \circ Y$ — NFA ops

$X \cap Y$, X^c — DFA ops

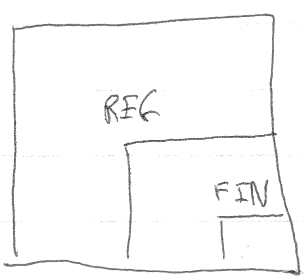
$\emptyset$, ε, c — primitive machines

$\emptyset$ 

ε 

c 

ALL



DFA = NFA

DFA $\subseteq$ NFA — decompiler

~~D~~DFA $\supseteq$ NFA — compiler

$L(DFA) \subseteq L(NFA)$

$\forall d \in DFA.\ \exists n \in NFA.\ L(d) = L(n)$

in: $d = (Q, \Sigma, q_0, \delta, F)$ $\qquad \delta: Q \times \Sigma \to Q$

out: $n = (Q', \Sigma, q_0', \delta', F')$ $\qquad \delta': Q' \times \Sigma_\varepsilon \to P(Q')$

$Q' = Q \qquad q_0' = q_0 \qquad F' = F$

$\delta'(q_i, c) = \{\ \delta(q_i, c)\ \}$

$\delta'(q_i, \varepsilon) = \emptyset$

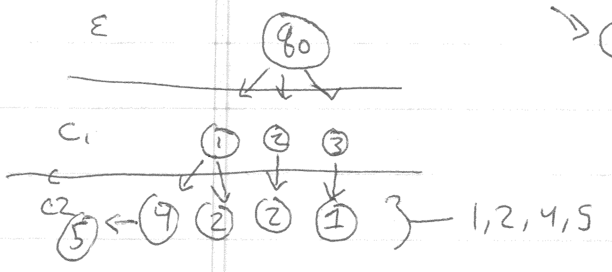
5-2

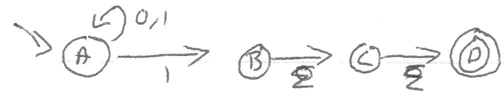
$L(NFA) \subseteq L(DFA)$

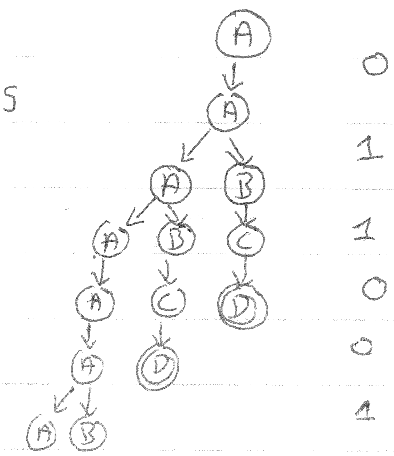
$\forall n \in NFA,\ \exists d \in DFA.\ L(n) = L(d)$

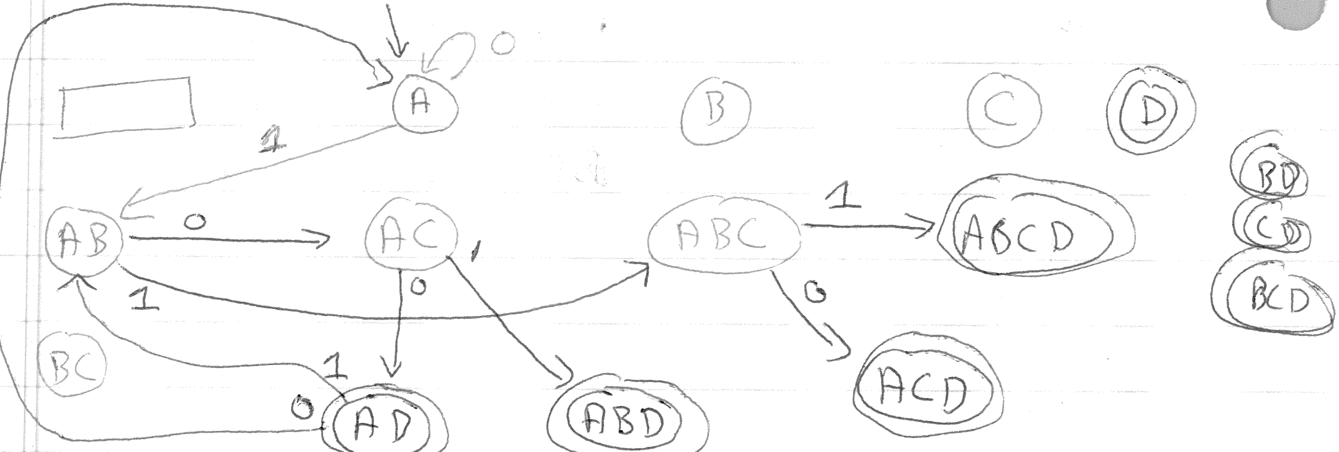
in: $n = (Q, \Sigma, q_0, \delta, F)$ $\quad \delta : Q \times \Sigma_\varepsilon \to P(Q)$

out: $d = (Q', \Sigma, q_0', \delta', F')$ $\quad \delta' : Q' \times \Sigma \to Q'$

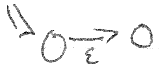








$Q' = P(Q)$ $\qquad q_0' = \{ q_0 \}$ $\qquad q_0' \in Q'$



F' = members of Q' that contain stuff in F

$\{ q_i' \in Q' \mid q_i' \cap F \neq \emptyset \}$

$\delta'(q_i', c) = q_j'$

$\{q_a, q_b, q_c, \ldots\}$ $\qquad \{q_x, q_y, q_z, \ldots\}$

For all states in input, look at δ, and combine together

5-3

$$\delta'(q_i', c) = \bigcup_{q_i \in q_i'} \delta(q_i, c)$$

$E : Q' \to Q'$ $\quad E(q_i')$ = the set of all places q_i''s elements could go follow epsilons, or not

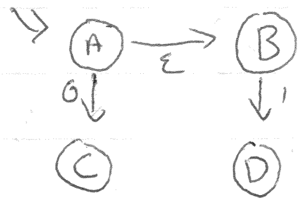
corrections:

$$q_0' = E(\{q_0\})$$

$$\delta'(q_i', c) = E\Big(\bigcup_{q_i \in q_i'} \delta(q_i, c)\Big)$$

$E(X)$ = l.f.p. of E_* s.t. X is contained

$$E_*(X) = X \cup \bigcup_{q \in X} \delta(q, \varepsilon)$$



$E_*(\{A\}) = \{A, B\}$ ✗

$E_*(\{A, B\}) = \{A, B\}$ ✓ fp

NFAs = DFAs

n-states $\Rightarrow$ 2^n - states (exp in memory)

Regular Expressions (REX)

$$r ::= r \cup r \mid r^* \mid r \circ r \mid r \cap r \mid r^c$$
$$\mid \emptyset \mid \varepsilon \mid c \in \Sigma$$

REX $\to$ NFA $\to$ DFA $\to$ run efficiently

5-4

$(0 \cup 1)^* \circ 1 \circ (0 \cup 1) \circ (0 \cup 1)$

$\Sigma^* \circ 1 \circ \Sigma \circ \Sigma$

" $.^* 1 ..$ "

D : ~~N~~FA $\rightarrow$ REX

in: n which is a k-state NFA

$D(n) = IN \circ RIP^k \circ OUT$

IN : k-state NFA $\rightarrow$ $(k+2)$-state GNFA

RIP : $(k+1)$-state GNFA $\rightarrow$ k-state GNFA

OUT : 2-state GNFA $\rightarrow$ REX

GNFA = "generalized" NFA where edges are REX

$(Q, \Sigma, q_a \in Q, \Delta, q_z \in Q)$

$\Delta : \underbrace{Q - \{q_z\}}_{\text{from}} \times \underbrace{Q - \{q_a\}}_{\text{to}} \rightarrow \underbrace{REX}_{\text{language of connecting strings}}$

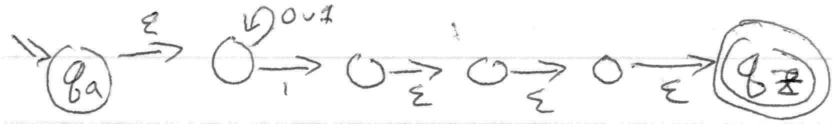
$\Delta(q_s, q_d) = r$ iff

$\forall x \in L(r) .\ q_s \overset{x}{\Rightarrow}{}^* q_d$ in NFA

IN = add states q_a and q_z

connect q_a to q_0 with epsilon

connect ~~q~~ $q_f \in F$ to q_z w/ ε



⋮

OUT = r

5-5)

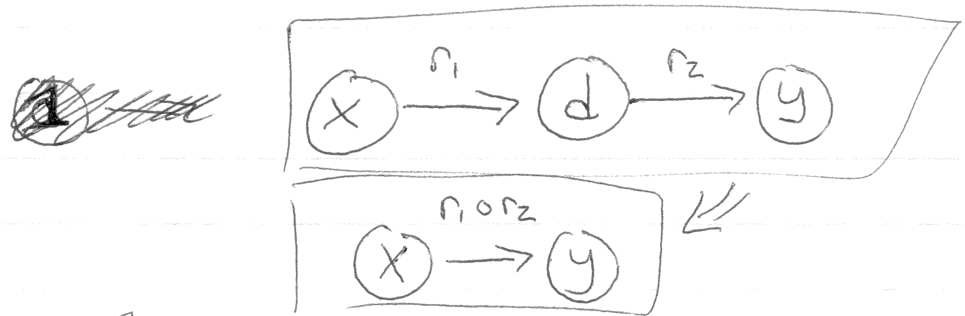
RIP : $(k+1)$-GNFA $\Rightarrow$ k-GNFA

in: $(Q, \Sigma, q_a, \Delta, q_z)$

out: $(Q - \{q_d\}, \Sigma, q_a, \Delta', q_z)$

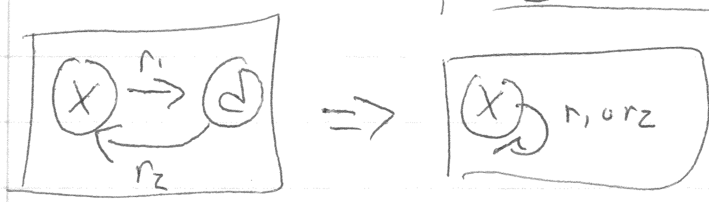
q_d = deadstate ($\neq q_a$ or q_z)

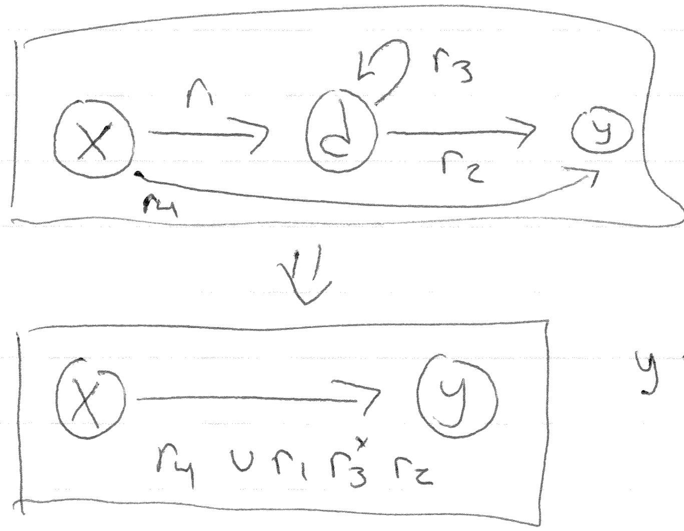
Δ' = update Δ to not mention q_d anymore



Δ

Δ'





y might be X