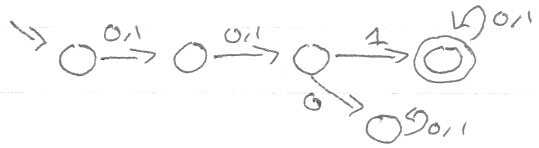
4-1

$3FE = \{ w \in \{0,1\}^* \mid$ 3rd character from the end of $w = 1 \}$

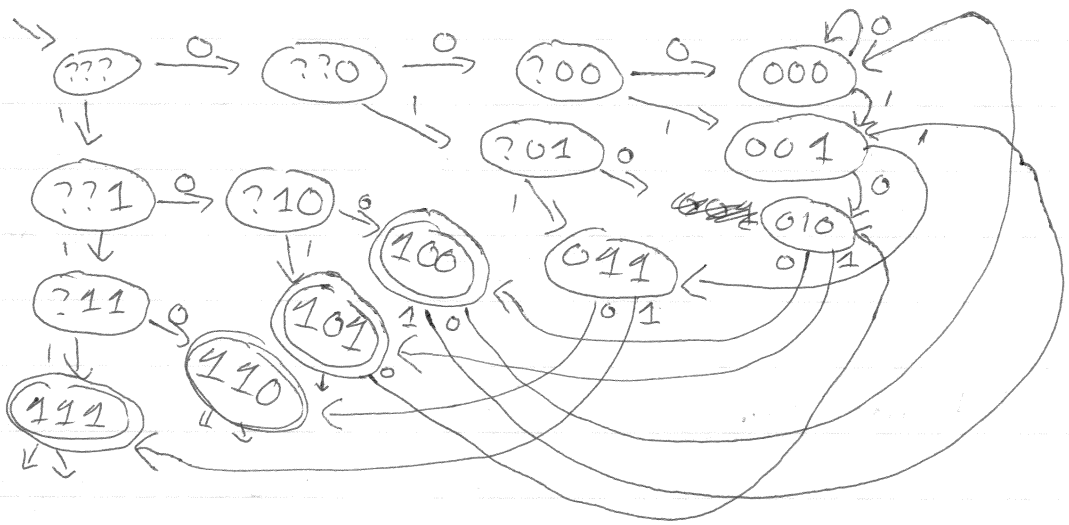
~~0100~~ $01\underline{1}00 \in 3FE$ $01\underline{0}00 \notin 3FE$

$3FS$ (start) $001000 \in 3FS$ $0111 \in 3FS$

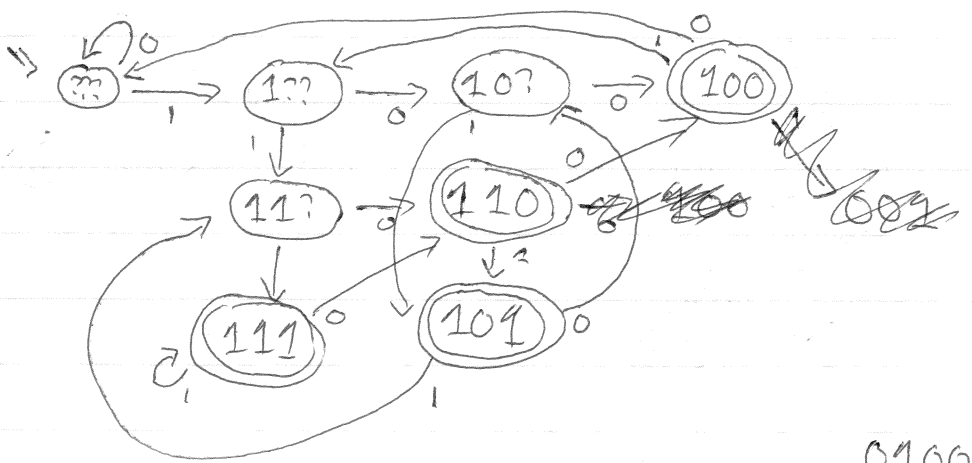
3FS



3FE



3FE

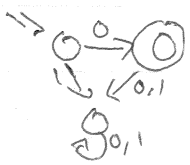


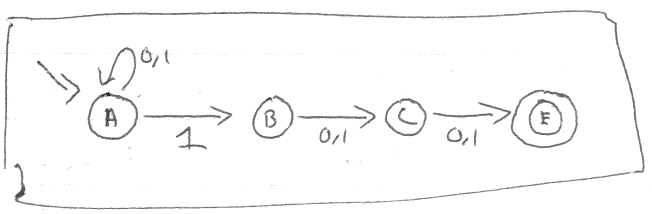
0100 ∈

11110 ∈

loop fwd

$(0 \cup 1)^* \circ 1 \circ (0 \cup 1) \circ (0 \cup 1) = 3FE$



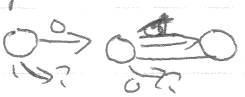


4-2

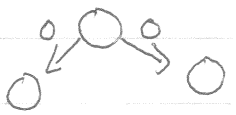
NFA – non-deterministic finite automata

DFA w/ some changes

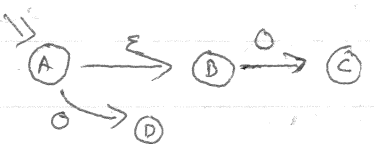
1) every input doesn't need an arrow/transitions



2) there may be multiple arrows for one input

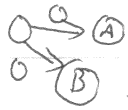


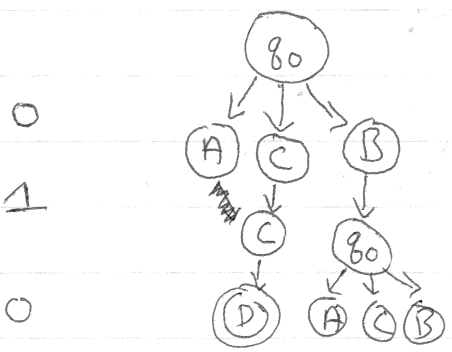
3) transition may be missing input (written ε) (epsilon)



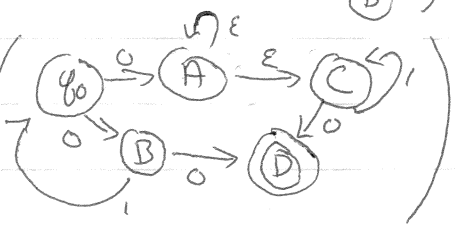
"0" → C or D

An NFA has <u>choices</u> – whether to read input (ε)
which transition to use





$010 \in L$ (



)

The language of the NFA is the set of strings where <u>there exists</u> a sequence of choices leading to an accepting state.

1) Oracle – read the future, make right choice
2) Back-tracking – DFS of the tree $O(1)$ mem $O(2^n)$ time
3) Forking / Parallel – BFS of the tree $O(2^n)$ mem $O(n)$ time
   DFS by level

4-3/

An NFA n is a 5-tuple of

$$(Q, \Sigma, q_0, \delta, F)$$

Q is a finite set of states

Σ is an alphabet

q_0 is $\in Q$

F is $\subseteq Q$

δ is ~~$Q \times \Sigma \longrightarrow Q$~~

$$\delta \text{ is } Q \times (\Sigma \cup \{\varepsilon\}) \longrightarrow P(Q)$$

sigma, left curly brace, epsilon

3FE

Q \ Σ	0	1	ε
A	A	B,A	$\emptyset$
B	C	C	$\emptyset$
C	E	E	$\emptyset$
E	$\emptyset$	$\emptyset$	$\emptyset$

$$L(n) = \{w \in \Sigma^* \mid w \text{ is accepted by } n\}$$

w is accepted by n iff

$q_0 \overset{w}{\Rightarrow}{}^* q_f$ s.t. $q_f \in F$

NFA n runs from q_i to q_j on w iff

$$\frac{}{q_i \overset{\varepsilon}{\Rightarrow}{}^* q_i} \qquad \frac{q_i \overset{a}{\Rightarrow} q_j \quad q_j \overset{w}{\Rightarrow}{}^* q_k}{q_i \overset{aw}{\Rightarrow}{}^* q_k} \qquad \text{~~$a \in \Sigma$~~ } a \in \Sigma \cup \{\varepsilon\}$$

NFA n steps from q_i to q_j on a iff

$$\frac{q_j \in \delta(q_i, a)}{q_i \overset{a}{\Rightarrow} q_j} \qquad a \in \Sigma \cup \{\varepsilon\}$$

~

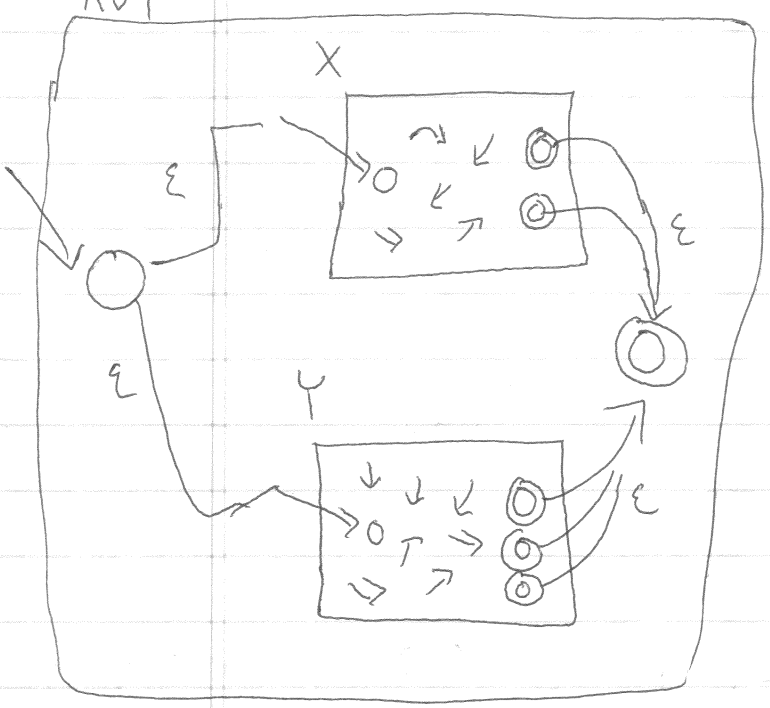
$$\frac{\delta(q_i, a) = q_j \qquad a \in \Sigma}{q_i \overset{a}{\Rightarrow} q_j}$$

$0100 = 0 \varepsilon 1 0 \varepsilon 0$

٩-٤

Suppose X is an NFA
and Y is an NFA

X∪Y



build X∪Y

X∘Y