Machine		Language Category	ALL
DFA	$\xrightarrow[L]{n \quad\quad 1}$	REG (regular)	$\supsetneq$ REG
(5-tuples)		(languages = sets of strings)	$\supsetneq$ FIN

$\xleftarrow{1 \quad\quad 1}$

find the smallest DFA (1021) whose language matches

$$\overset{2}{\{a,b\}} \cup \overset{2}{\{b,c\}} = \overset{3}{\{a,b,c\}}$$

$$\{a,b\} \uplus \{b,c\} = \{(0,a)\ (0,b)\ (1,b)\ (1,c)\}$$

Union

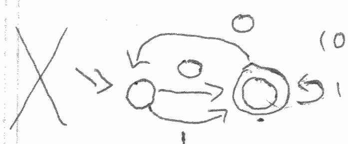
$a \in X \cup Y$ iff $a \in X$ or $a \in Y$

[odd length]

[ends in 1]

$L(\cdot) = X \in REG$ $\quad\quad$ $L(?) = Y \in REG$

$\exists d_3 .\ L(d_3) = X \cup Y$? If so, then $X \cup Y \in REG$



(odd or ends in 1)

0100

REG is closed under union

$$\boxed{\forall X \in REG, Y \in REG,\ \exists A \in REG.\ A = X \cup Y}$$

$X \in REG$ iff $\exists d_X \in DFA.\ L(d_X) = X$

$$\boxed{\forall d_X \in DFA, d_Y \in DFA,\ \exists d_A \in DFA.\ L(d_A) = L(d_X) \cup L(d_Y)}$$

Input: $d_X : DFA = (Q_X, \Sigma, q_{0X}, \delta_X, F_X)$ $\quad\quad$ $\exists a \in U.\ P(a)$

$\quad\quad d_Y : DFA = (Q_Y, \Sigma, q_{0Y}, \delta_Y, F_Y)$ $\quad\quad$ constructive: give a, Prove P(a)

Output: $d_A : DFA = (Q_A, \Sigma, q_{0A}, \delta_A, F_A)$ $\quad\quad$ $\neg \forall a \in U.\ \neg P(a)$

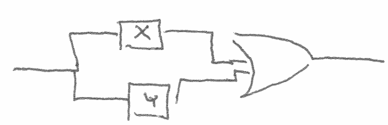
$Q_A = Q_X \times Q_Y$

$q_{0A} = (q_{0X}, q_{0Y})$

$\delta_A : Q_A \times \Sigma \to Q_A = (Q_X \times Q_Y) \times \Sigma \to (Q_X \times Q_Y)$

$$\delta_A((q_X, q_Y), a) = (\delta_X(q_X, a), \delta_Y(q_Y, a))$$

$$F_A = \cancel{\underbrace{F_X \times F_Y}_{n}} \quad \underbrace{(F_X \times Q_Y)}_{X \text{ accepts}} \cup \underbrace{(Q_X \times F_Y)}_{Y \text{ accepts}}$$



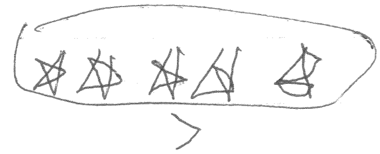
Regular Languages are closed under the Regular Operations

Union : $\cup$ (we proved)
Intersect: $\cap$ (we proved =. $F_X \times F_Y$)
Complement: ^{c} ($F_A = F_X^c$, we proved)
Reversal : ^{R}
Star : *
Concat : $\circ$

$\left.\begin{array}{l} \text{Reversal} \\ \text{Star} \\ \text{Concat} \end{array}\right\}$ — hard proofs

Road map:

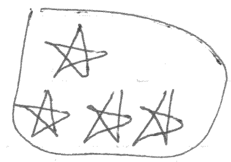
concat is hard on DFAs



Define BlahFA

prove concat on BlahFA (very easy)

Show that BlahFA = DFA (medium)



We like BlahFA!

Use BlahFA to make DFA-PL

$\boxed{\text{Talk about smallest Union}}$