

23-1)

① $A_{TM} \in \Sigma_1$

② $A_{TM} \notin \Sigma_0$

③ $P \in \Sigma_1 \wedge \bar{P} \in \Sigma_1 \Rightarrow P \in \Sigma_0$

$A_{TM} \notin \Sigma_1$

Mapping Reducibility

If $A \rightarrow B$ and $B \notin \Sigma_0$, then $A \notin \Sigma_0$

$E_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset \}$

$E_{TM} \in \Sigma_0$

① $E_{TM} \rightarrow A_{TM}$

Assume $M_e \in \Sigma_0$ that solves E_{TM}
Build $M_a \in \Sigma_0$ that solves A_{TM}

$M_e : \langle M_2 \rangle \rightarrow Y/N$ if $L(M_2)$ is \emptyset

$M_a : \langle M_1, w \rangle \rightarrow Y$ if $w \in L(M_1)$

M_a : On input $\langle M_1, w \rangle \dots$

Run $M_e(\langle M_2 \rangle)$

$M_e(\langle M_1 \rangle) = Y$ then $L(M_1) = \emptyset$

$M_e(\langle M_1 \rangle) = N$ then $L(M_1) \neq \emptyset$

M_2 : On input x ,

Simulate M_1 on w

If accept, then accept

o.w., reject

$M_e(\langle M_2 \rangle) = N \rightarrow w \in L(M_1)$

$= Y \rightarrow w \notin L(M_1)$

$L(M_2) \neq \emptyset \rightarrow w \in L(M_1)$

$ALL_{TM} \notin \Sigma_0$

$EQ_{TM} = \{ \langle M_1, M_2 \rangle \mid M_1, M_2 \in TM, L(M_1) = L(M_2) \}$

$EQ_{TM} \rightarrow A_{TM}$

Assume M_g decides EQ_{TM}

$EQ_{TM} \rightarrow E_{TM}$

Build an A_{TM} decider

$E_{TM} \rightarrow A_{TM}$

Build an E_{TM} decider - M_e

$M_e : \langle M_3 \rangle$

$L(M_1) = L(M_2)$ iff $L(M_3) = \emptyset$

call $M_g(\langle M_1, M_3 \rangle)$

M_1 : On input x , reject

$L(M_1) = \emptyset$

3-2/

$$REG_{TM} = \{ \langle M \rangle \mid M \in L(TM) \text{ , } L(M) \in REG \}$$

Assume M_1 decides REG_{TM}

Build M_2 decides ATM

M_2 : On input $(\langle M_1, w \rangle)$

Run $M_1(\langle M_2 \rangle)$

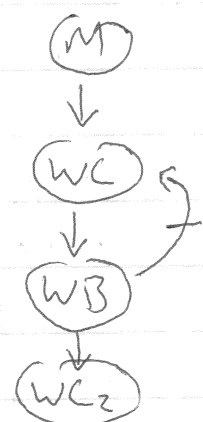
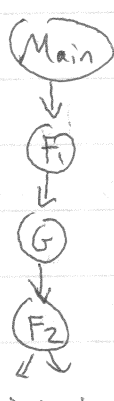
$L(M_2) \in REG$ iff $w \in L(M_1)$

M_2 : On input x ,

Simulate M_1 on w

If accept, accept x

If reject, check if x is $0^n 1^n$, if so, yes
o.w., NO



23-3/

LBA - linear-bounded automata

= A TM w/o an infinite tape
ie. w/ a finite tape

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$$

$$c_0 = \sqcup [q_0] \sqcup \sqcup$$

No rule for adding blanks

$$\begin{bmatrix} a \\ a \\ z \end{bmatrix} \in \Gamma$$

A_{DFA}, A_{CFG}, E_{DFA}, E_{CFG}, $0^n 1^n 0^n, \dots \in \text{LBA}$

LBAs are weaker than TMs and stronger than CFGs

ALBA is decidable (by an LBA)

$$\text{ALBA} = \{ \langle M, w \rangle \mid M \text{ is an LBA and } w \in L(M) \}$$

On input, $\langle M, w \rangle$:

Simulate M on w for X steps

if accept, accept, o.w. reject

Suppose that $|w|$ is N , $\Gamma^{|w|} = \Gamma^N$ different tapes

$\times N$ different head positions

$$\# \text{ of configs} = |Q| \times |w| \times |\Gamma|^{|w|}$$

$\times |Q|$ different control states

$$|w|=10, |\Gamma|=3, |Q|=4 = 4 \times 10 \times 3^{10}$$

$< X$

ELBA not decidable

Date	Description	Particulars	Amount
2020-01-01	Balance b/d		1000
2020-01-05	Cash	100	100
2020-01-10	Bank	200	200
2020-01-15	Sales	300	300
2020-01-20	Purchases	150	150
2020-01-25	Expenses	50	50
2020-01-31	Total		1000
2020-02-01	Balance c/d		1000
2020-02-05	Cash	100	100
2020-02-10	Bank	200	200
2020-02-15	Sales	300	300
2020-02-20	Purchases	150	150
2020-02-25	Expenses	50	50
2020-02-28	Total		1000
2020-03-01	Balance b/d		1000
2020-03-05	Cash	100	100
2020-03-10	Bank	200	200
2020-03-15	Sales	300	300