23-1 ① $A_{TM} \in \Sigma_1$ ② $A_{TM} \notin \Sigma_0$ ③ $P \in \Sigma_1 \wedge \bar{P} \in \Sigma_1 \Rightarrow P \in \Sigma_0$

$\overline{A_{TM}} \notin \Sigma_1$

Mapping Reducibility

If $A \to B$ and $B \notin \Sigma_0$, then $A \notin \Sigma_0$

$E_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset \}$

$E_{TM} \notin \Sigma_0$ ① $E_{TM} \to A_{TM}$

↳ Assume $M_e \in \Sigma_0$ that solves E_{TM}
Build $M_a \in \Sigma_0$ that solves A_{TM} ←

$M_e : \langle M_2 \rangle \to$ Y/N if $L(M_2)$ is $\emptyset$

$M_a : \langle M_1, w \rangle \to$ Y if $w \in L(M_1)$

M_a : On input $\langle M_1, w \rangle$ ...

Run $M_e(\langle M_2 \rangle)$

$M_e(\langle M_1 \rangle) =$ Y then $L(M_1) = \emptyset$

$M_e(\langle M_1 \rangle) =$ N then $L(M_1) \neq \emptyset$

M_2 : On input x,

Simulate M_1 on w

If accept, then accept

o.w., reject

$M_e(\langle M_2 \rangle) =$ N $\Rightarrow w \in L(M_1)$

$= \cancel{\text{Y}} \Rightarrow w \notin L(M_1)$

$L(M_2) \neq \emptyset \Rightarrow w \in L(M_1)$

$ALL_{TM} \notin \Sigma_0$

$EQ_{TM} = \{ \langle M_1, M_2 \rangle \mid M_1, M_2 \in TM, L(M_1) = L(M_2) \}$

R) $EQ_{TM} \to A_{TM}$

$EQ_{TM} \to E_{TM}$

$E_{TM} \to A_{TM}$

Assume M_q ~~solves~~ decides EQ_{TM}

~~Build an A_{TM} decider~~

Build an E_{TM} decider — M_e

$M_e : \langle M_3 \rangle$

Call $M_q(\langle M_1, M_3 \rangle)$

$L(M_1) = L(M_3)$ iff $L(M_3) = \emptyset$

M_1 : On input x, reject $L(M_1) = \emptyset$

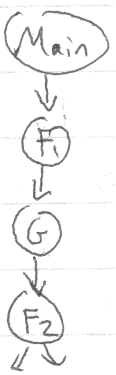
3-2

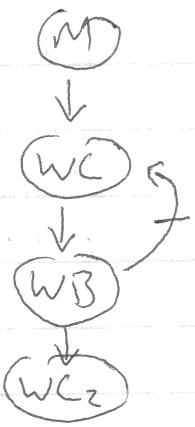
$REG_{TM} = \{ \langle M \rangle \mid M \in L(TM) \text{ , } L(M) \in REG \}$

Assume M_r decides REG_{TM}
Build M_a decides A_{TM}

M_a : On input $(\langle M_1, w \rangle)$
Run $M_r(\langle M_2 \rangle)$ $L(M_2) \in REG$ iff $w \in L(M_1)$

M_2 : On input x,
Simulate M_1 on w
If accept, accept x
If reject, check if x is 0^n1^n, if so, yes
o.w., NO





23-3/

LBA – linear-bounded automata

= A TM w/o an infinite tape
ie. w/ a finite tape

$\delta : Q \times \Gamma \Rightarrow Q \times \Gamma \times \{L, R\}$

$c_0 = \sqcup [q_0] w \sqcup$

No rule for adding blanks

$\in \Gamma$

$A_{DFA}, A_{CFG}, \Sigma_{DFA}, E_{CFG}, 0^n1^n0^n, \ldots \in LBA$

LBAs are weaker than TMs and stronger than CFGs

A_{LBA} is decidable (by an LBA)

$A_{LBA} = \{ \langle M, w \rangle \mid M \text{ is an LBA and } w \in L(M) \}$

On input, $\langle M, w \rangle$:
Simulate M on w for X steps
if accept, accept, o.w. reject

Suppose that $|w|$ is N, $\Gamma^{|w|} = \Gamma^N$ different tapes

$\times N$ different head positions

$\times |Q|$ different control states

$\#$ of configs $= |Q| \times |w| \times |\Gamma|^{|w|} < X$

$|w| = 10, |\Gamma| = 3, |Q| = 4 \quad = 4 \times 10 \times 3^{10}$

E_{LBA} ~~no~~ decidable