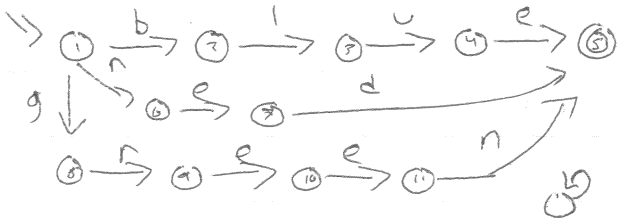
2-1

A = { blue, red, green }

= program in DFA

= computer

= set



DFA - deterministic finite automata

○ - a state (element of some set)

⇒○ - the start state

◎ - an accepting state

(p) —a→ (q) - a transition

yellow ∈ A? X

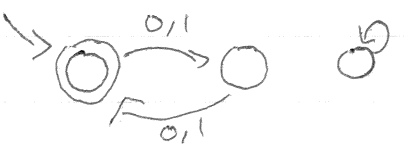
red ∈ A? ✓

digit tree / trie / binary trie — datastructures for sets

{ saw, hacksaw }

EB = { w ∈ {0,1}* | |w| = 2n i.e. w is even length }

ε ∈ EB 01 ∈ EB 0001 ∈ EB 011 ∉ EB



EPKS = { w ∈ {Charmander, Squintle, BUlbasaur}* | |w|=2n }


```
int c=0
while (getc() ≠ EOF)
    c++
return  c%2 == 0
```

A DFA d is a 5-tuple =

$$(Q, \Sigma, q_0, \delta, F) \qquad \forall \Sigma$$

Q = the finite set of states
Σ = an alphabet (a finite set)
$q_0 \in Q$ = the start state
$F \subset Q$ = the accepting states (a set)
$\delta : Q \times \Sigma \Rightarrow Q$ = the transition function
in: current char, current state; out: next state

Need a mapping between DFAs and sets

L (language) : DFA $\Rightarrow P(\Sigma^*)$

$$L(d) = \{ w \in \Sigma^* \mid w \text{ is accepted by } d \}$$

A string w is accepted by d iff

$$q_0 \overset{w}{\Rightarrow}{}^* q_f \quad \text{and} \quad q_f \in F$$

A DFA d runs from q_i to q_j on $w \in \Sigma^*$ $(q_i \overset{w}{\Rightarrow}{}^* q_j)$ iff

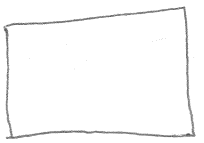
$$q_i \overset{\varepsilon}{\Rightarrow}{}^* q_i \qquad \frac{q_i \overset{a}{\Rightarrow} q_i \quad q_i \overset{w}{\Rightarrow}{}^* q_k}{q_i \overset{aw}{\Rightarrow}{}^* q_k}$$

A DFA d steps from q_i to q_j on $a \in \Sigma$ $(q_i \overset{a}{\Rightarrow} q_j)$ iff

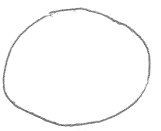
$$\delta(q_i, a) = q_j$$

2-3/

What languages have a DFA?



ALL = every possible set of strings $= P(\Sigma^*)$



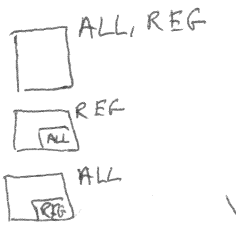
DFA = every possible DFA

REG = every language that has a DFA = the regular languages

1) ALL = REG

2) ALL $\subsetneq$ REG

3) REG $\subsetneq$ ALL

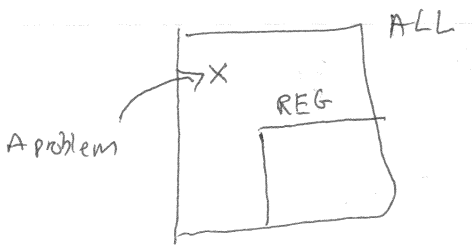


X^* ✓

X ✗

✓

There is a problem that a DFA can't solve



$0^n 1 0^m 1 0^{n+m}$

00100010000 ✓

0101000 ✗

FIN = the set of finite languages

FIN $\subsetneq$ REG

Closure

$\subseteq P(U)$

A set X is closed under operation $O : X \Rightarrow U$

iff

$\forall x \in X.\ O(x) \in X$

$\mathbb{N}$, add 1	Pokemon, evolve
$\mathbb{Z}$, sub 1	Presentations, reverse
movies, mute	movie, frame X

What operations is REG closed under?

$O : X \ X \Rightarrow U$

$\forall x, y \in X.\ O(x, y) \in X$

---

reactive (emacs) → Transducers = DFA + $\mathcal{O}$ (output fuction)

batch (gcc) → DFAs

1) Add output alphabet, Γ
2) $\mathcal{O} : Q \Rightarrow \Gamma$

or 2') $\delta' : Q \times \Sigma \Rightarrow Q \times \Gamma$

UML Statecharts