REG $\subset \Sigma_0$

Every DFA can be turned into a decider

$Q' = Q \cup \{A, R\}$

$q_0' = q_0$

$\delta'(q_i, a) = (\delta(q_i, a), a, R)$

$\delta'(q_i, \sqcup) = (A, a, R)$ if $q_i \in F$

(R, a, R) o.w.

Language $A_{DFA} = \{ \langle D, w \rangle \mid D \text{ is a DFA and } w \in L(D) \}$

$\langle\rangle$ = magical encoding brackets

$\langle (\{a,b\}, \{\sqcup, \not\sqcup\}, a, \begin{array}{c|cc} & a & b \\ \hline \sqcup & b & a \\ \not\sqcup & b & a \end{array}, \{a\}), \sqcup\not\sqcup\not\sqcup\sqcup \rangle \in A_{DFA}$

$A_{DFA} \in \Sigma_0$ (cool!)

tape 0 : $\langle D, w \rangle$

tape 1 : $\langle w \rangle$

tape 2 : $\langle q_0 \rangle$

$\Rightarrow^*$

$w = w_0 \ldots w_n$

$\langle D, w \rangle$

$\langle w_i \ldots w_n \rangle$

$\langle q_i \rangle$

$q_0 \overset{w_0 \ldots w_{i-1}}{\Rightarrow} q_i$

~~int fac (int x) {~~

~~return foo (x+1);~~

```
int fac (int n, int a) {
  if (n == 0) return a;
  else return fac(n-1, n*a);
}
... fac ( 4096, 1);
```

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$A_{NFA} = \{ \langle N, w \rangle \mid N \text{ is an NFA and } w \in L(N) \}$

$A_{NFA} = A_{DFA} \circ$ NFA $\Rightarrow$ DFA compiler

$A_{REX} = A_{NFA} \circ$ REX $\Rightarrow$ NFA

"Meta-Programs" Programs about Programs is input/output

- compiler
- interpreter
} okay

- syntax checker
- linters
- pretty-print
} boring

= analyzer } exciting

$E_{DFA} = \{ \langle D \rangle \mid D \text{ is a DFA and } L(D) = \emptyset \}$

No path from q_0 to $q_f \in F$

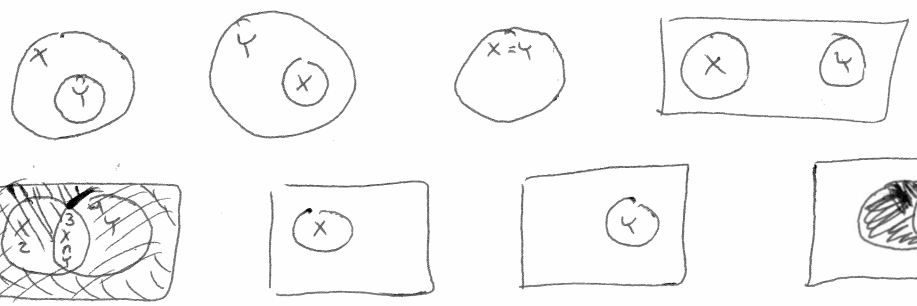
Depth-first search

DFA is closed under union, complement, concat

$X \cup Y$ and $X = \emptyset$ → Y

$X \circ Y$ and $X = \emptyset$ → $\emptyset$

$EQ_{DFA} = \{ \langle X, Y \rangle \mid X \in DFA, Y \in DFA, L(X) = L(Y) \}$



$\neg X \cup \neg Y$ $\quad$ $X - Y$ $\quad$ $\neg X$ $\quad$ $X \cap \neg Y = X - Y$

$X \cap Y$ $\quad$ $Y - X$ $\quad$ $\neg Y$ $\quad$ $Y \cap \neg X = Y - X$

$X = Y \Rightarrow (X \cap \neg Y) \cup (Y \cap \neg X) = \emptyset$

19-3/

A_X = interpreter for X

E_X = emptiness checker

EQ_X = equality

$A_{CFG} \in \Sigma_0$?

If we assume G is CNF, then we know that a derivation of w has at most $|w|$ levels, thus $2^{|w|}$ nodes, thus we only check $2^{|w|}+1$ options.

$E_{CFG} \in \Sigma_0$?

Bottom-up dynamic program to computes paths from variables to strings of terminals

$EQ_{CFG} \notin \Sigma_0$