

Non-Determinism

Intuition /  $\delta$  / Semantics / Galois Connection

"Go left or right"

which ever works"

$$\delta: Q \times \Gamma \rightarrow P(Q \times \Gamma \times \{L, R\})$$

$$P(Q) \times \{L, R\}$$

$$\delta: Q \times \Gamma \rightarrow (Q \times \Gamma \times \{L, R\})$$

$$\times (Q \times \Gamma \times \{L, R\})$$



original)  $\delta(q_i, b) = (q_j, L, L)$

left  $ua[q_i]bv \Rightarrow u[q_j]acv$

new  $(q_i, L, L) \in \delta(q_i, b)$

Galois Connection



f ↓

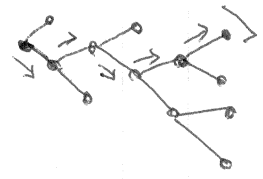


$$f = \text{concrete } c$$

$$f \circ x = a(g(c(x)))$$

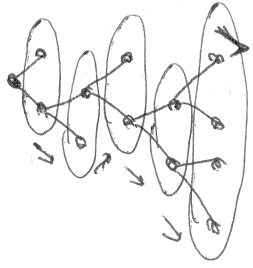
non-det

oracle

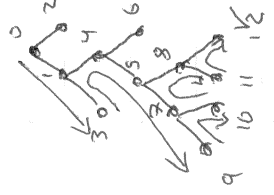


X

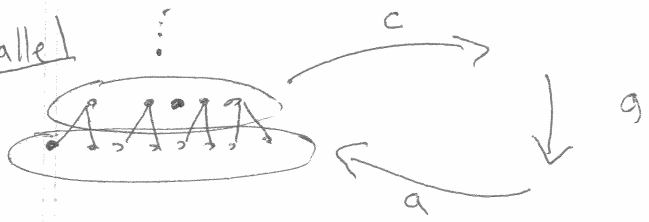
forking/parallel



back tracking



17-2 / Parallel



$$A = \{ \text{configs} \} = P(\Gamma^* \times Q \times \Gamma^*)$$

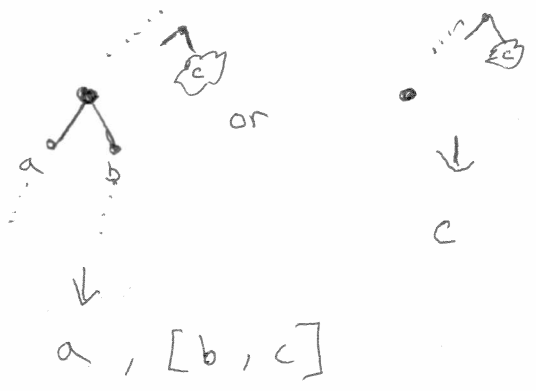
$$ua[q_i]bv = (ua, q_i, bv) \in (\Gamma^*, Q, \Gamma^*)$$

$$c \left( \{ ua[q_i]bv, x\alpha[q_i]\beta y \} \right) = [u]ua[q_i]bv \# x\alpha[q_i]\beta y \#$$

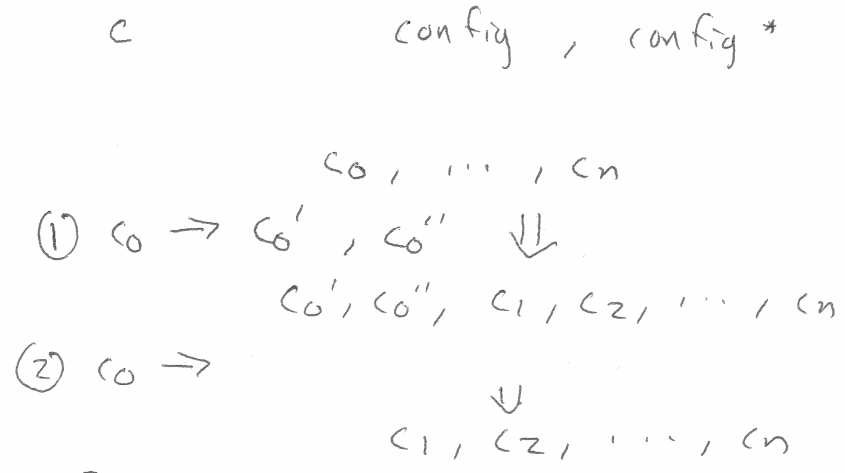
$$\{ [ \cdot ], \#, \cup \} \cup Q \cup \Gamma \subseteq \Gamma'$$

g = for each config  
consult  $\delta$   
copy config w/ change  
or die

Back-tracking



where we are  
where we will go in future  
(continuation)



- 0  $\rightarrow$  1, 2  $\rightarrow$  3, 4, 2  $\rightarrow$
- 4, 2  $\rightarrow$  5, 6, 2  $\rightarrow$  7, 8, 6, 2  $\rightarrow$
- ~~8, 6, 2  $\rightarrow$  9, 10, 8, 6, 2  $\rightarrow$~~
- 10, 8, 6, 2  $\rightarrow$  8, 6, 2  $\rightarrow$  11, 12, 6, 2  $\rightarrow$
- (12), 6, 2  $\rightarrow$   $\checkmark$

17-3/

Deciders

vs

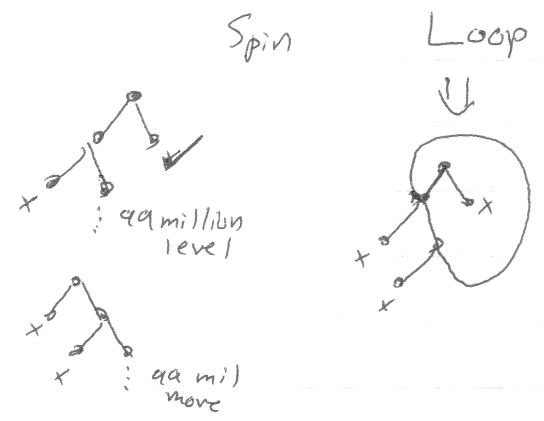
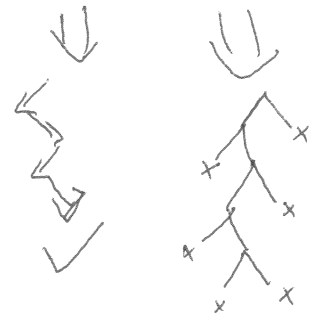
Recognizers

$\Sigma_0$

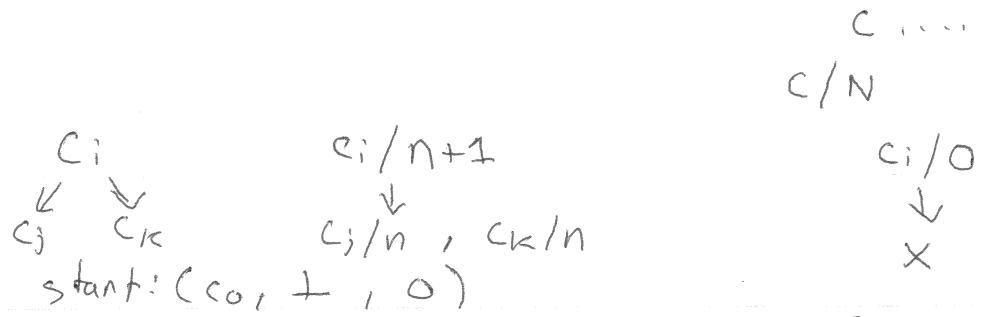
$\Sigma_1$

$\{YES, NO\}$

$\{Y, N, D\}$



Parallel — copies (spin or loop) take space  
 Back-Tracking — (input  $c_0$ , current stack  $\uparrow$ ,  $n$ )



$(c_0, \perp, n) \rightarrow (c_0, [c_0/n], n+1)$   
 $(c_0, [c_i/n+1, \dots], m) \rightarrow (c_0, [c_i/n, c_k/n, \dots], m)$   
 $\rightarrow (c_0, [\dots], m)$

