$$\delta : Q \times \Gamma \to Q \times \Gamma \times \underset{\text{left, right}}{\{L, R\}}$$

"Stay-Put" TM

$$\delta' : Q \times \Gamma \to Q \times \Gamma \times \{L, R, \overset{\text{stay}}{S}\}$$

old $\dfrac{\delta(q_i, b) = (q_j, c, L)}{ua[q_i]bv \Rightarrow u[q_j]acv}$

new $\dfrac{\delta(q_i, b) = (q_j, c, S)}{\underset{\text{not needed}}{ua}[q_i]bv \Rightarrow ua[q_j]cv}$

$$\begin{tikzcd}
\text{STM} \arrow[r, Rightarrow, "n^*"] \arrow[d, "\text{compiler}"] & \text{final config} \\
\text{TM} \arrow[ur, Rightarrow, "m^*"'] &
\end{tikzcd}$$

	STM	NTM
	c_0	c_0
never uses stay	$\Downarrow_*$	$\Downarrow_*$
	c_i	c_i
	$\Downarrow$ stay	$\Downarrow_*$
	c_{i+1}	c_{i+1}
may use stay	$\Downarrow_*$	$\Downarrow_*$
	c_n	c_n

$\boxed{\forall a \in \Gamma, \exists q_* \in Q}$

$$\dfrac{\text{left } \dfrac{\delta(q_i, b) = (q_*, c, L)}{ua[q_i]bv \Rightarrow u[q_*]acv} \qquad \dfrac{\delta(q_*, a) = (q_j, a, R)}{u[q_*]acv \Rightarrow ua[q_j]cv} \text{ right}}{ua[q_i]bv \Rightarrow^* ua[q_j]cv}$$

stay : $(\overset{\text{states}}{N}, \overset{\text{step per stay}}{1})$

normal : (N + # stay states , 2 steps per stay)

Normalizing Program

input $= n_0 \;\dots\; n_m$

where $n_i = b_0 \dots b_7$

output $= n'_0 \;\dots\; n'_m$

$$n'_i = n_i \Big/ \sum_{j=0}^{m} n_j$$

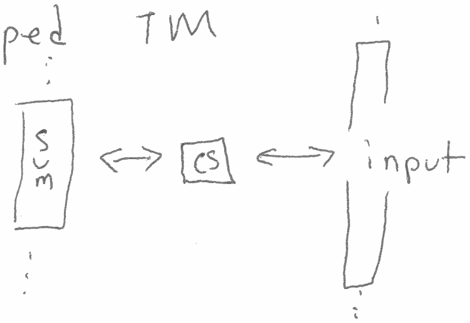
$$\dots \sqcup\, [\text{start}]\; 01000 \cent 00\underset{\boxed{0}}{\overset{\uparrow}{1}}11\,0 \;\dots\; \sqcup \dots$$

$\Gamma = \sqcup, 0, 1, ☺$

$\boxed{0}, \boxed{1}, ⟨0⟩, ⟨1⟩$

$= a, b, c, d, e, f$

2-taped TM



$$\delta : Q \times (\Gamma \times \Gamma) \to Q \times \big((\Gamma \times \{L, R, S\}) \times (\Gamma \times \{L, R, S\}) \big)$$

2-configuration : 2 tapes, 2 heads, 1 state

$$\begin{matrix} u_0 \\ u_1 \end{matrix} \Big[q_i \Big] \begin{matrix} v_0 \\ v_1 \end{matrix} \qquad C_0 = \begin{matrix} \varepsilon \\ \varepsilon \end{matrix} \Big[q_0 \Big] \begin{matrix} \sqcup \\ \varepsilon \end{matrix} \qquad C_a = \begin{matrix} u_0 \\ u_1 \end{matrix} \Big[q_a \Big] \begin{matrix} v_0 \\ v_1 \end{matrix}$$

2-step semantics $\Rightarrow_2$

1-step $\Rightarrow$ now $\Rightarrow_1$

$$\frac{u_0 [q_i] v_0 \Rightarrow_1 u'_0 [q_j] v'_0 \qquad u_1 [q_i] v_1 \Rightarrow_1 u'_1 [q_j] v'_1}{\begin{matrix} u_0 \\ u_1 \end{matrix} \Big[q_i \Big] \begin{matrix} v_0 \\ v_1 \end{matrix} \Rightarrow_2 \begin{matrix} u'_0 \\ u'_1 \end{matrix} \Big[q_j \Big] \begin{matrix} v'_0 \\ v'_1 \end{matrix}}$$

$\delta_{T1}(q_i, b) = (q_j, c, d)$

if $\cancel{\delta(q_i, b) = (q_j}$

$\delta(q_i, (b, *)) = (q_j, (c, d), (*, *))$

simulation of 2 by 1

2TM | **NTM**

$$C_0 = \begin{matrix} \varepsilon \\ \varepsilon \end{matrix}\left[q_0 \right]\begin{matrix} w \\ \varepsilon \end{matrix}$$

$$\Downarrow_*$$

$$\begin{matrix} ua \\ x\alpha \end{matrix}\left[q_i \right]\begin{matrix} bv \\ \beta y \end{matrix} \xrightarrow{\text{concretization}} [q_i]\, ua\boxed{b}v \blacksquare x\alpha\boxed{\beta}y$$

Galois connections

$$\Downarrow \qquad\qquad\qquad \Downarrow_*$$

$$\begin{matrix} u'a' \\ x'\alpha' \end{matrix}\left[q_j \right]\begin{matrix} b'v' \\ \beta' y' \end{matrix} \xleftarrow{\text{abstraction}} [q_j]\, u'a'\boxed{b'}v' \blacksquare x'\alpha'\boxed{\beta'}y'$$

$$\delta(q_i, (b, \beta)) = (q_j, (b', d_1), (\beta', d_2))$$

0 n 1 m 2 5 q 4 p 3 o

2-taped machine

K-taped machine (multi-tape Turing machine)