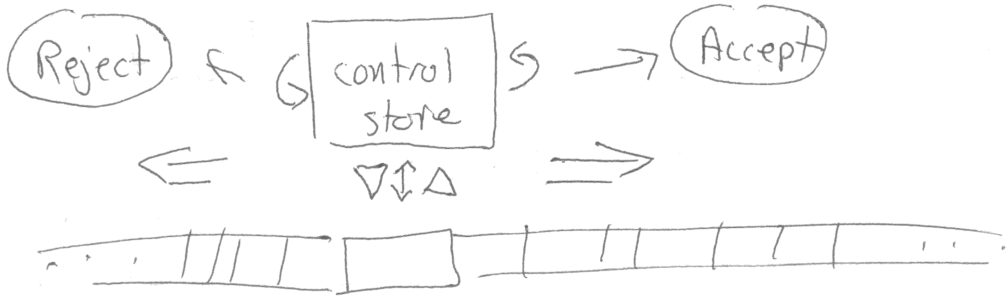
15-1



A Turing Machine (TM) $\dagger$ is a 7-tuple

$(Q, \Sigma, \Gamma, \delta, q_0, q_a, q_r)$

Q = a finite set of states

Σ = a finite alphabet (input) $\quad \sqcup \notin \Sigma$

Γ = a finite alphabet (tape) $\quad \Sigma \subset \Gamma \quad \sqcup \in \Gamma$

δ = transition function

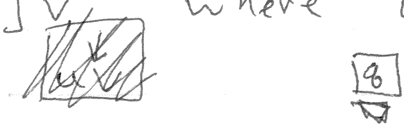
$$Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$$

$q_0 \in Q$ - start $\quad q_a \in Q$ - accept $\quad q_r \in Q$ - reject

$Q - \{q_a, q_r\}$

A configuration of TM $\dagger$ is

$u[q]v$ where $u, v \in \Gamma^*$ $\quad q \in Q$



$u_0 \;,, \; u_{n-1} u_n v_0 v_1 v_2 v_3 \ldots v_n$

The initial configuration of TM $\dagger$ on input $w \in \Sigma^*$ is

$[q_0]w$ $\quad$ (ALT: $\sqcup^x [q_0] w \sqcup^y$) $\quad x, y \in \mathbb{N}$

The language of a TM $\dagger$ is

$\{ w \in \Sigma^* \mid [q_0] w \Rightarrow^* u[q_a]v \}$

~~steps~~ runs

A config $u[q_i]v$ runs to $x[q_j]y$

$$u[q_i]v \Rightarrow^* x[q_j]y$$

refl $\quad u[q_i]v \Rightarrow^* u[q_i]v$

step1 $\quad \dfrac{a[q_j]b \Rightarrow^* x[q_k]y \qquad u[q_i]v \Rightarrow a[q_j]b}{u[q_i]v \Rightarrow^* x[q_k]y}$

LT

blankL $\quad u[q_i]v \Rightarrow^* \sqcup u[q_i]v$

blankR $\quad u[q_i]v \Rightarrow^* u[q_i]v\sqcup$

A config $u[q_i]v$ <u>steps</u> to $x[q_j]y$

$$u[q_i]v \Rightarrow x[q_j]y$$

left $\quad \dfrac{\delta(q_i, b) = (q_j, c, L)}{ua[q_i]bv \Rightarrow u[q_j]acv}$ $\qquad q_i, q_j \in Q \quad u, v \in \Gamma^*$ $\qquad a, b, c \in \Gamma$

right $\quad \dfrac{\delta(q_i, b) = (q_j, c, R)}{u[q_i]bv \Rightarrow uc[q_j]v}$

A trace of TM t is a string of configurations
$c_0 \ldots c_n$
such that c_0 is the initial (ie $[q_0]w$ for some w)
and $c_i \Rightarrow^* c_j$ for all $i, j \in N$ $i \leq j$

A TM t accepts w if a trace exists where
$c_0 = [q_0]w$ and $c_n = u[q_a]v$

reject w = $\neg$ accept w

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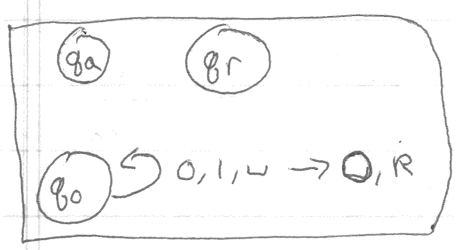
$[q_0]w \Rightarrow^* u_a[q_a]v_a$ (accept)

$\Rightarrow^* u_r[q_r]v_r$ (reject)

$\Rightarrow^* u_d[q_d]v_d \Rightarrow \ldots$ (diverge)

never reaches q_a / q_r

DIVERGE



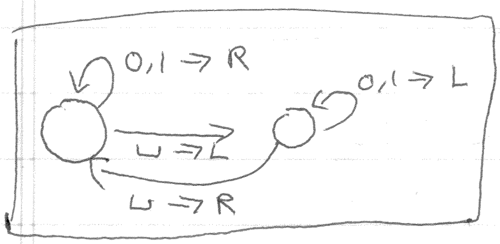
$\Rightarrow^+ = \Rightarrow^*$ w/o refl

Diverge: 1. LOOP
2. SPIN

1. LOOP = $[q_0]w \Rightarrow^* u_\ell[q_\ell]v_\ell$
$\Rightarrow^* u_\ell[q_\ell]v_\ell$ (not via refl)

2. SPIN = for all
$u_i[q_i]v_i \Rightarrow^+ u_j[q_j]v_j$
~~$u_i \ne u_j$ ...~~ $\neg(u_i = u_j \land q_i = q_j \land v_i = v_j)$

LOOP



Accepted = reach accept
Not be accept = reach reject OR diverge

Σ_1 Turing-recognizable ::= a set of languages A (like REG, CFL)
where $\exists t \in TM \quad L(t) = A$

$\boxed{A \in \Sigma_1 \text{ iff } \exists t.\ L(t) = A}$

Σ_0 Turing-decidable
$A \in \Sigma_0$ iff $\exists t,\ L(t) = A$ AND t is a decider

A TM t is a decider if $\forall w \in \Sigma^*.\ [q_0]w \Rightarrow^* u_a[q_a]v_a$
(never diverges) or $\Rightarrow^* u_r[q_r]v_r$

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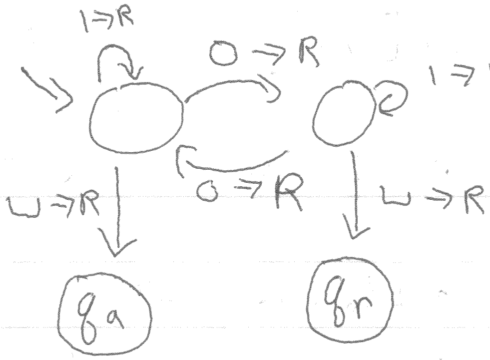
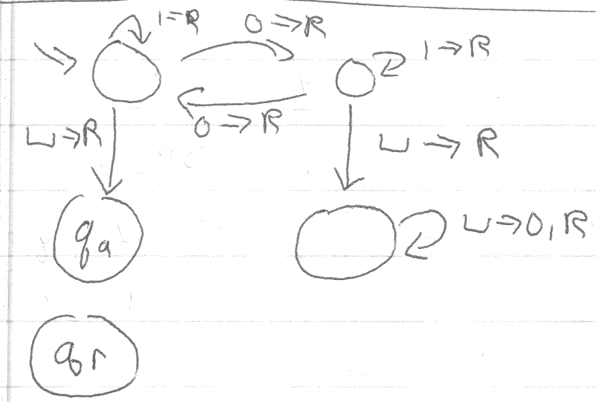


even # of 0s $\in \Sigma_0$ is a decider



is $\underbrace{\text{not a decider}}$, → recognizer

Any machine is a recognizer

ALL

REG

CFL

Σ_1

Σ_0

$\Sigma_0 \subset \Sigma_1$

REG $\subset$ ALL

CFL $\subset$ ALL

$\Sigma_1 \subset$ ALL

$\Sigma_0 \subset$ ALL

REG $\subset$ CFL

CFL $\not\subset$ REG (i.e. $0^n 1^n$)

$\Sigma_1 \not\subset$ CFL (i.e. w#w)

$\Sigma_1 \subset \Sigma_0$? i.e. is there a language with no decider?

ALL $\subset \Sigma_1$? i.e. is there a language with no TM?

Transducer TM = computable function $[q_0]w \Rightarrow^*_u [HALT]v$

means $f(w) = v$ total = "decider" partial = diverge

Enumerator $[q_0] \Rightarrow^* u_0[PRINT]v_0 \Rightarrow^* u_1[P]v_1 \Rightarrow^* \ldots$

$v_0 \in$ ACCEPTED $v_1 \in$ ACCEPT