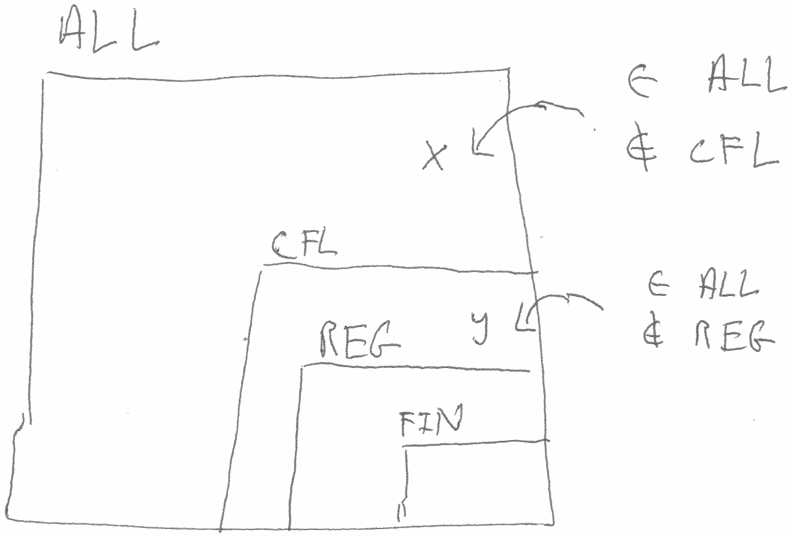
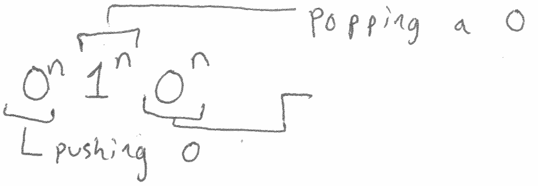
12-1



$0^n 1^n$

$S \to \varepsilon \mid 0S1$



$0^n 1^i 0^j$ $i+j=2n$

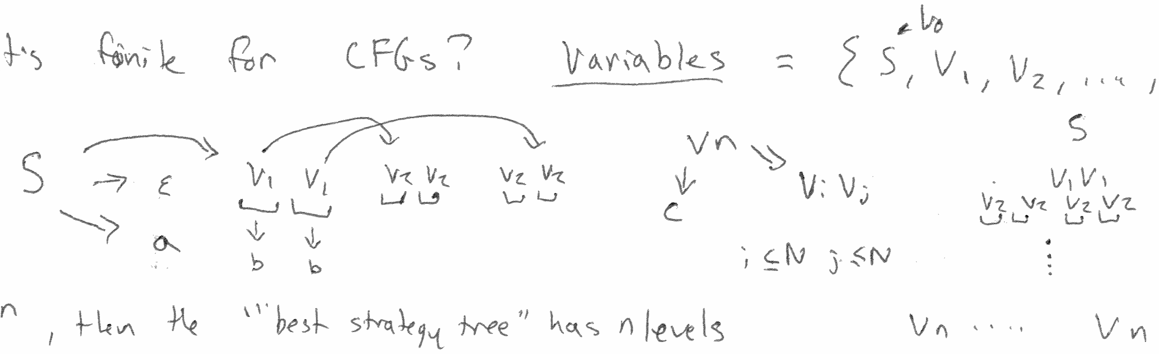
cool property P (regular pumping property)

$\forall A \in REG.\ P(A) \quad \wedge \quad \neg P(0^n 1^n) \implies 0^n 1^n \notin REG$

Idea of RPP := "DFAs have loops" ⇐ finite # of states + every step uses one

+ "If w is accepted and on loop path ⇒ infinite accepted"

What's finite for CFGs? Variables = $\{ S, V_1, V_2, \dots, V_n \}$

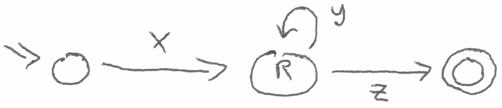


$|w| = 2^n$, then the "best strategy tree" has n levels

$|w| > 2^n$, then there's n+1 levels ($n = |V|$)

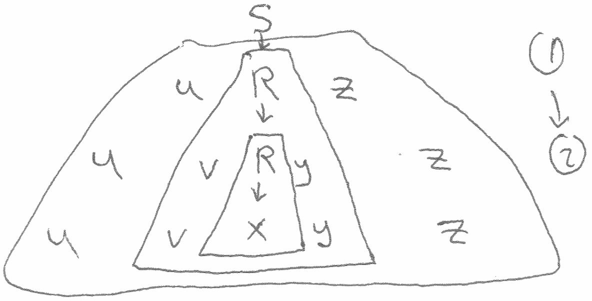
$S \to^* uRz \to^* uvRyz \to^* uvxyz$

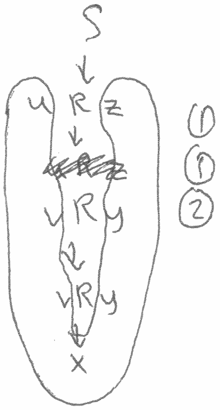
$u, z \in \Sigma^*$ $R \in V$ $v, y \in \Sigma^*$ $x \in \Sigma^*$



$uvxyz \in A \quad |\cdot| > 2^{|V|}$

12-2





$uvvxyyz$

$R \xrightarrow{①} vRy$

$R \xrightarrow{②} x$

$uv^ixy^iz \in A$

$|vxy| < 2^{|v|}$ $|vy| > 0$

$E \to 0 \mid 1 \mid E+E$

$\ulcorner n \urcorner = \ulcorner n-1 \urcorner + \ulcorner n-1 \urcorner$

$\ulcorner 0 \urcorner = 1$

$s = \ulcorner p \urcorner$

$x = \ldots$ $v = \varepsilon$ $x = 1$ $y = +1$ $z = \ldots$

Context-Free Pumping Property

$\forall A \in CFL,$

$\exists p \in \mathbb{N},$ // $p = 2^{|v|} + 1$

$\forall (s \in A \mid |s| \ge p),$

$\exists (u,v,x,y,z \in \Sigma^* \mid s = uvxyz \wedge |vxy| \le p \wedge |vy| > 0),$

$\forall i \in \mathbb{N},$

$uv^ixy^iz \in A$

0^n1^n

$s = 0^p1^p$

$u = 0^{p-1}$ $v = 0$ $x = \varepsilon$ $y = 1$

$z = 1^{p-1}$

$0^{p+i}1^{p+i}$

¬CFPP

$\exists A \in ALL,$

$\forall p \in \mathbb{N}.$

$\exists (s \in A \mid |s| \ge p)$

$\forall (u,v,x,y,z \in \Sigma^* \mid s = uvxyz \wedge |vxy| \le p \wedge |vy| > 0),$

$\exists (i \in \mathbb{N}),$

$uv^ixy^iz \notin A$

12-3/

$A = 0^n 1^n 0^n$ $a^n b^n c^n$

given: p

choose $s = 0^p 1^p 0^p$ $(|s| \geq p?$ $3p \geq p)$ ✓

given: u, v, x, y, z $s = uvxyz$

$|vxy| \leq p$ $|vy| > 0$

choose: i

case 1 (vxy has 1 thing in it)	case 2 (it has two things)	case 3 (3 things
vxy is all in first 0s (a)	vxy is 1st 0s and the 1s (ab)	$vxy = 0 1^p 0$
the 1s (b)	1s and 2nd 0s (bc)	$\lvert vxy \rvert = p+2$
the 2nd 0s (c)		×

1a. $u = 0^h$ $vxy = 0^j$ $z = 0^k 1^p 0^p$ $h+j+k = p$

$0^j = 0^{\hat{v}} 0^{\hat{x}} 0^{\hat{y}}$ $\hat{\alpha} = |\alpha|$ $j = \hat{x} + \hat{v} + \hat{y}$

$uv^i x y^i z = 0^h 0^{\hat{v} i} 0^{\hat{x}} 0^{\hat{y} i} 0^k 1^p 0^p \in A$

$\Leftarrow h + \hat{v} i + \hat{x} + \hat{y} i + k = p \Leftarrow \hat{v} i + \hat{y} i = \hat{v} + \hat{y} \Leftarrow i = 1$

2ab $u = 0^h$ $vxy = 0^j 1^k$ $z = 1^l 0^p$

case 2ab.1 = $v = 0^j$ $x = \varepsilon$ $y = 1^k$ case 2ab.2= $v=0^{\hat{v}}$ $x=0^{\hat{x}_0} 1^{\hat{x}_1}$ $y=1^{\hat{y}}$

..... as $i \uparrow$, we add 0s and 1s, but 0s at end

$0^{p+k} 1^{p+k} 0^p \notin A.$