Theory of Computation

- In math, what is a computer?
  "algorithm"
  "program"
  "procedure"
- what can they do? / What can't they do?

Different kinds of computers

A computer is a set of ???
A kind of computer is a set of computers (i.e set of sets)

A set is ... "list of things"
{Charizard, Blastoise, Pikachu} representations of sets
{P, B, C}

All sets with written representation as list of things
= Finite = FIN (FIN2187)

How to write down non-finite sets?
wanted: TRUE = set of all true math statements

A set X is a function $\in$ from elements to Bool
membership function

$X \cup Y$ is a set where $a \in X \cup Y$ iff $a \in X$ or $a \in Y$
union

backwards $\in$

intersect : $X \cap Y \ni a$ iff $a \in X$ and $a \in Y$

complement : X^c $\bar{X}$ $a \in X^c$ iff $a \notin X$ and $a \in \mathcal{U}$ universe

$\emptyset$ empty set (mt) $\forall a,\ a \notin \emptyset$

1-2/ subset $X \subset Y$ iff $\forall a.\ a \in X \Rightarrow a \in Y$

(n-)Tuple, but focus on 2-tuple (pair)

If X is a ~~Pair~~ pair of A and B, then

$\pi_0(X)$ is an A (7, "charizand")

$\pi_1(Y)$ is a B is a pair of $\mathbb{N}$ and Pokemon

set-builder notation

$X \times Y = \{ (x,y) \mid x \in X \text{ and } y \in Y \}$

cartesian product ↗ such that ↗

$(a,b) \in X \times Y$ iff $a \in X$ and $b \in Y$

Relation R on X, Y, and Z is a subset of $X \times Y \times Z$. (3 place relation)

pluses $\subset \mathbb{N} \times \mathbb{N} \times \mathbb{N}$

$$\frac{}{\text{pluses } 0\ 0\ 0}$$

$$\frac{\text{pluses } N\ M\ Q}{\text{pluses } (1+N)\ M\ (1+Q)}$$

A function f from X to Y, write $f : X \to Y$ is a relation on X and Y where

$\forall x, y_1, y_2.\ f\ x\ y_1 \wedge f\ x\ y_2 \Rightarrow y_1 = y_2$ and

reflexive: $\forall x.\ R\ x\ x$

symetric: $\forall x, y.\ R\ x\ y \Rightarrow R\ y\ x$

trans: $\forall x, y, z.\ R\ x\ y \wedge R\ y\ z \Rightarrow R\ x\ z$

Powerset of X written $\mathbb{P}(X)$ or 2^X

$a \in \mathbb{P}(X)$ iff $a \subset X$

$\{0,1\}$ $\mathbb{P}(\ )$ $= \{ \overset{00}{\emptyset}, \overset{01}{\{0\}}, \overset{10}{\{1\}}, \overset{11}{\{0,1\}} \}$

If X is size N then 2^X is size 2^N

Strings

A string w is a finite sequence of Σ

Σ is a finite set called an alphabet

$|w|$ is the length of w — Σ sigma

w_i is the ith thing (Σ) in w — $\in$ member

ε (epsilon) the string of length 0 — '' '' ε epsilon

w^R w reverse, $123^R = 321$

$x \circ y$ / xy — concatenation

$|xy| = |x| + |y|$ $(xy)_i = x_i$ if $i < |x|$

$y_{i-|x|}$ o.w.

w^n is n copies of w $(|w^n| = n \cdot |w|)$

w^* (kleene star) = any number of copies of w

NOT a string. A set of strings.

$u \in w^*$ = first kind of | computer we'll

iff $u = w^n$ for some n

"language of Σ" sets of strings of Σ

1-4

Computers are sets of strings (language)

A "problem" is also a language

$\Sigma = \{0, 1, +, =\}$

$\{ x+y=z \mid x,y,z \in \{0,1\}^*$ and
$x+y=z \}$

= binary ~~tree~~ plus~~-~~ Set
true plus set

Computer's goal is to "recognize" elements of the set.