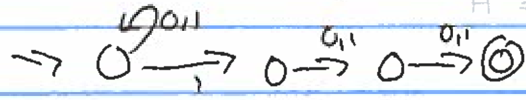
5-1

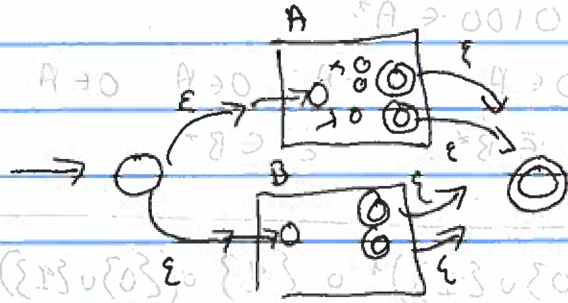
NFAs

transitions now could go multipleplaces
(equiv to having ε - (epsilon) transition)



= Anything ∘ 1 ∘ Two things

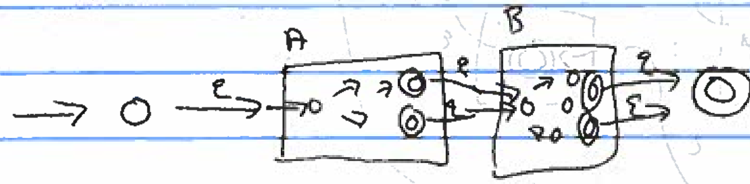
DFA ∪ = n^2 states



= A∪B

NFA∪ = 2n + 2

(A∪B)∪C



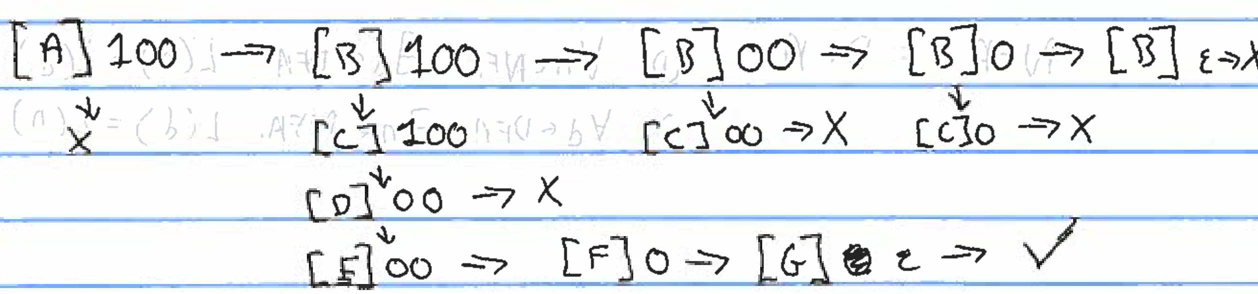
2n+2

(0∪1) ∘ (0∪1)

(Anything ∘ (1) ∘ Two things)

1 is 3rd from end 

100 ✓ 1100 ✓ 0001 X



Anything = (0∪1) repeated

5-2 Kleene star A^* $A^* = \{\varepsilon\} \cup A \circ A^*$

$w \in A^*$ iff $w = w_0 \circ w_1 \circ \dots \circ w_n$

where $w_i \in A$

$A = \{0\} \cup \{1\}$ $0100 \in A^*$

$0 \in A$ $1 \in A$ $0 \in A$ $0 \in A$

$B = \{011\}$ $011\,011\,011 \in B^*$ $\varepsilon \in B^*$

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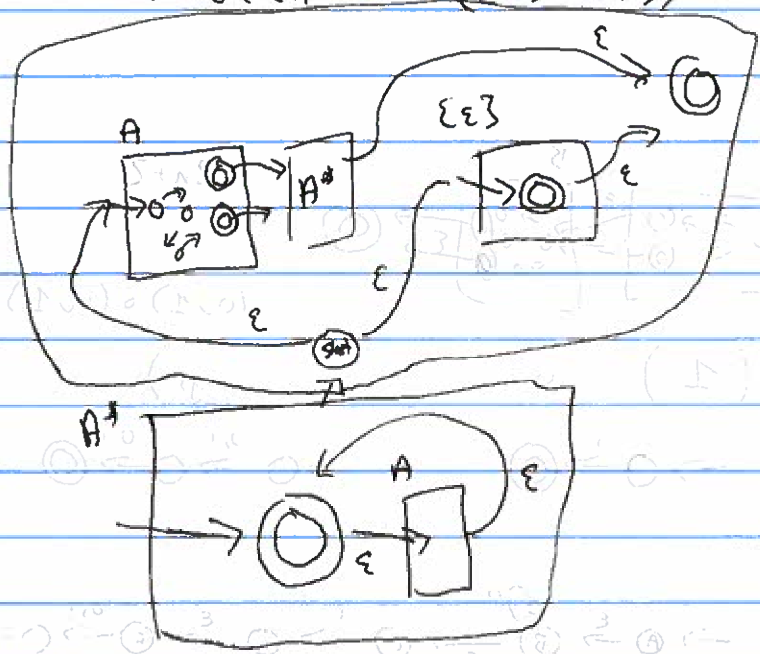
1 is 3rd char from end $= (\{0\} \cup \{1\})^* \circ \{1\} \circ (\{0\} \cup \{1\}) \circ (\{0\} \cup \{1\})$



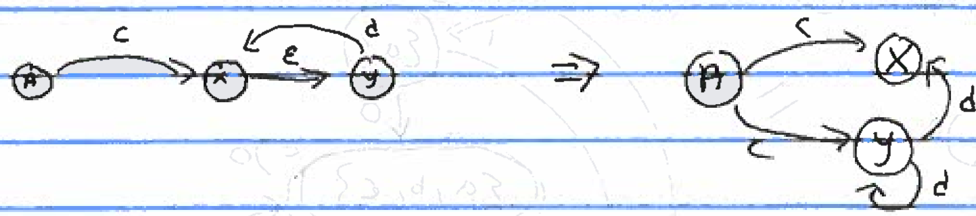
Regular languages = those defined by DFAs

NFA = DFA ① $\forall n \in NFA, \exists d \in DFA, L(n) = L(d)$

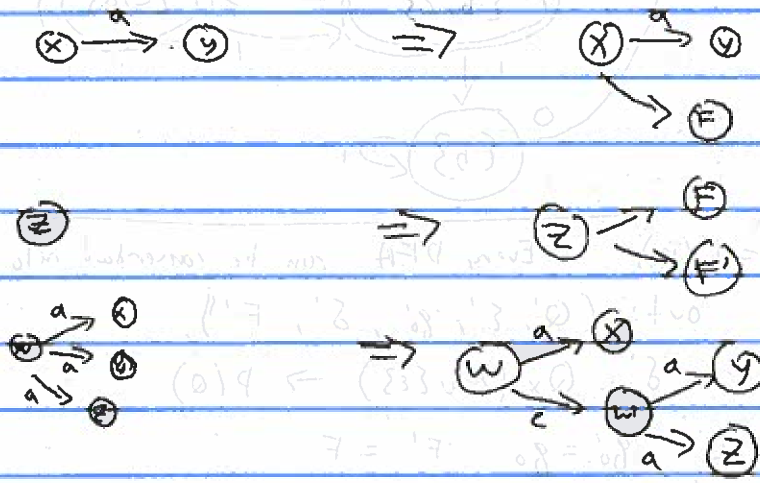
② $\forall d \in DFA, \exists n \in NFA, L(d) = L(n)$

5-3

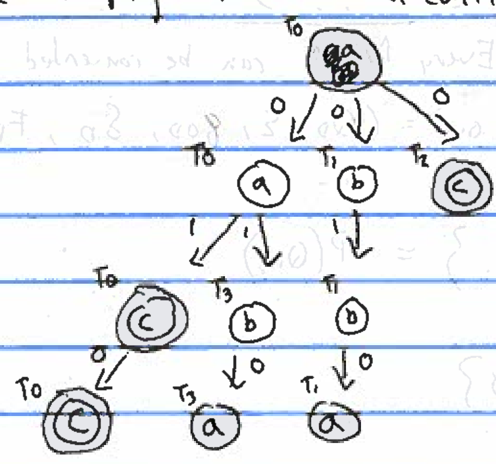
Imagine all NFAs didn't have ε-transitions



Imagine all NFA transitions go to exactly two places



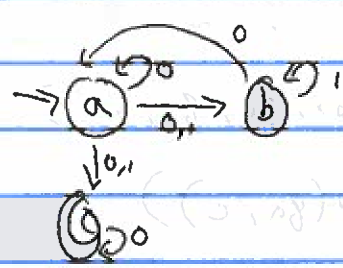
These imply ... a config tree



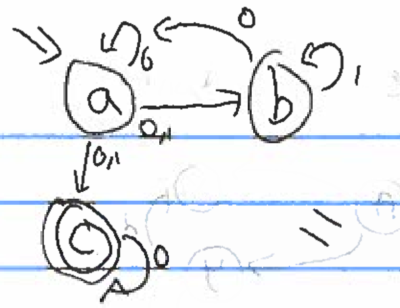
$T\emptyset = [a]\,010$

$T_0\ [a]\,10$ $T_1\ [b]\,10$ $T_2\ [c]\,10$

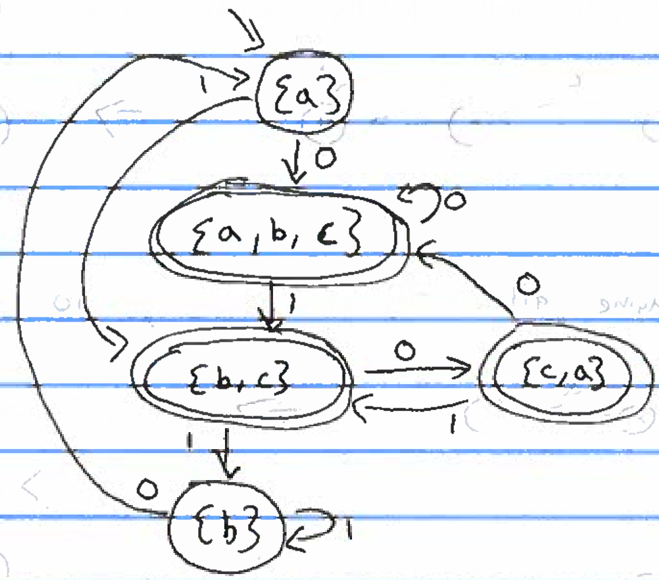
$T_0\ [c]\,0$ $T_3\ [b]\,0$ $T_1\ [b]\,0$ ×



5-4



=



abc

100

111

011 101

010

② $\forall d \in DFA. \exists n \in NFA. L(d) = L(n)$ "Every DFA can be converted into an NFA"

in: $(Q, \Sigma, q_0, \delta, F)$ out: $(Q', \Sigma', q_0', \delta', F')$

$\delta: Q \times \Sigma \Rightarrow Q$ $\delta': Q \times (\Sigma' \cup \{\varepsilon\}) \to P(Q)$

$Q' = Q$ $\Sigma' = \Sigma$ $q_0' = q_0$ $F' = F$

$\forall x \in Q'. \ \delta'(x, \varepsilon) = \emptyset$

$\forall x \in Q'. \ \forall c \in \Sigma'. \ \delta'(x, c) = \{\delta(x, c)\}$

① $\forall n \in NFA. \exists d \in DFA. L(n) = L(d)$ "Every NFA can be converted into a DFA"

in = $(Q_N, \Sigma, q_{0N}, \delta_N, F_N)$ out = $(Q_D, \Sigma, q_{0D}, \delta_D, F_D)$

$Q_D = \{\emptyset, Q_N, \{q_{0N}\}, \{a,b\}, \ldots\} = P(Q_N)$

$q_{0D} = E(\{q_{0N}\})$

$F_D = \{q_i \in Q_D \mid q_i \cap F_N \neq \emptyset\}$

$\delta_D(q_i \in \underset{P(Q_N)}{\underset{\shortparallel}{Q_D}}, c \in \Sigma) = q_j \in Q_D$

$q_j = \{q_x, q_y, q_z, \ldots\}$

$q_i = \{q_a, q_b, q_c, \ldots\}$ $= \bigcup_{q_a \in q_i} E(\delta_N(q_a, c))$

$E: Q_D \to Q_D =$ $E(A) = A \cup \bigcup_{a \in A} \delta_N(a, \varepsilon)$ compute fixed-point of E