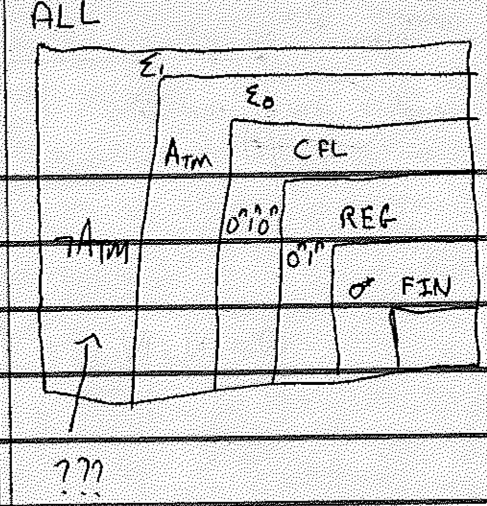
23-1



$ALL = \Sigma_1$ — FALSE

$A \in \Sigma_0$ iff $A \in \Sigma_1$ and $\neg A \in \Sigma_1$

$\Rightarrow$ — obvious normal negated

$\Leftarrow$ — not obvious

assume X is the TM for $A \in \Sigma_1$

Y is the TM for $\neg A \in \Sigma_1$

Halting Problem! =   construct Z, the TM for $A \in \Sigma_0$

$U := A_{TM} \in \Sigma_1$ $A_{TM} \notin \Sigma_0$ $Z(w) :=$

Assume $\boxed{\neg A_{TM} \in \Sigma_1}$ is false

$\therefore$ $A_{TM} \in \Sigma_0 \Rightarrow$ False

take a step in $X(w)$

take a step in $Y(w)$

repeat but if X says $Y \to Y$

Y says $Y \to N$

$\neg A_{TM} = \{ \langle M, w \rangle \mid M \in TM \text{ and } w \notin L(M) \}$

When are two sets the same size?

cardinality : set $\Rightarrow$ num $|A| = |B|$ iff $card(A) = card(B)$

cardinality $(0^*) = ?\ \aleph_0$ Georg Cantor

Set A is the same size as Set B $(A \approx B)$ iff

exists $f : A \to B$ such that $\forall b \in B.\ \exists a \in A.\ f(a) = b$

and

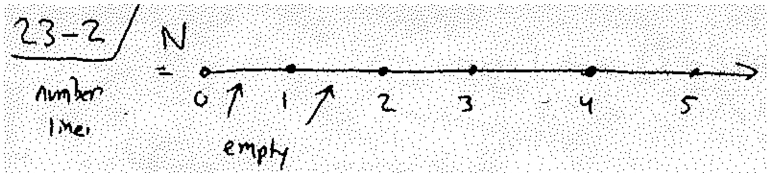
$\forall a, a' \in A, \forall b.\ f(a) = b \wedge f(a') = b \Rightarrow a = a'$

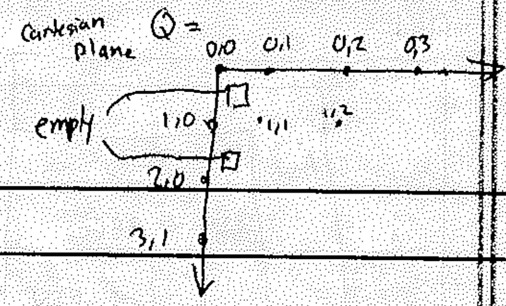
Nat $(N) \not\approx$ Evens (wrong)

$A = N$ $B = $ Evens $f(n) = 2 \times n$ $\Rightarrow N \approx$ Evens

B is countable $:= N \approx B$ $|B| = \aleph_0$ $\aleph_0$

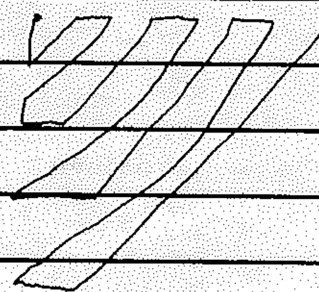
23-2





$N \not\approx Q = N \times N$

wrong!



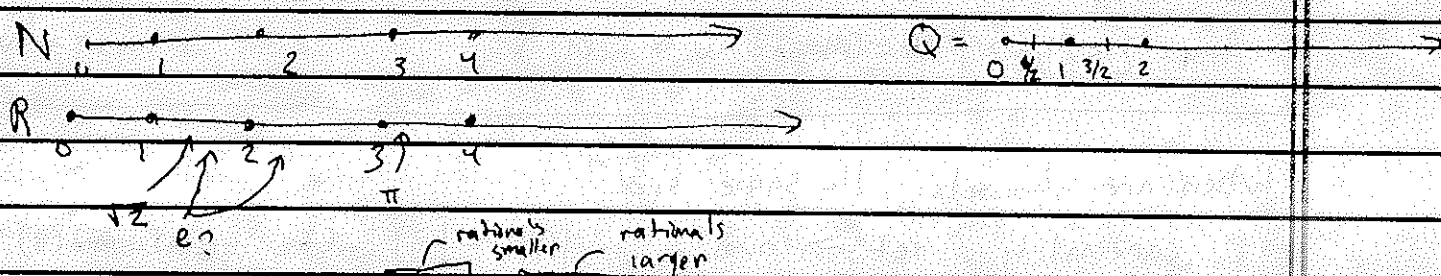
$Q = N \times N$

$N \approx N \times N$

$f(x,y) = \frac{1}{2}(x+y)(x+y+1) + y$

$N \approx N \times N \times N \qquad g(x,y,z) = f(x, f(y,z))$

$N \approx N^k$ for all $k \in N$



Dedenkind Cut $\quad R = P(Q) \times P(Q)$

Cauchy Sequence $\quad R = N \to Q \quad$ s.t. $f(n)$ approaches the r as $n \to \infty$

Infinite Digit Sequences $\quad R = N \to D$ (where $D = [0,9]$)

Infinite Binary Sequence $\quad N \to \{0,1\} \qquad R_{[0,1)} \qquad x \in R_{[0,1)} \qquad x = 0.01010----$

$\frac{1}{4} + \frac{1}{16}$

$\pi/4$ = some fun $\qquad f(n) = $ if $(n == 1)$ then 1

$(n == 3)$ then 1

o.w. $\emptyset$

$N \not\approx R_{[0,1)}$ (IBS)

Assume $N \approx IBS, \Rightarrow f(n:N) : IBS$ (and f is onto / one-to-one)

$\forall b \in B. \exists a \in N. f(a) = b$ — onto

$\neg(\forall b \in B. \exists a \in N. f(a) = b) = \exists b \in IBS. \forall a \in N. \boxed{f(a) \neq b} = \exists i \in N. f(a)(i) \neq b(i)$

$b(i) := \neg f(i)(i) \quad \Rightarrow N \not\approx IBS \Rightarrow N \not\approx R$

Diagonalization Argument

3-3

$$\left.\begin{array}{c} ALL \\ \approx \\ IBS \end{array}\right\} \text{hard} \qquad \not\approx \qquad \Sigma_1$$

$\approx$ ← every lang in Σ_1 is some TM (by defn)

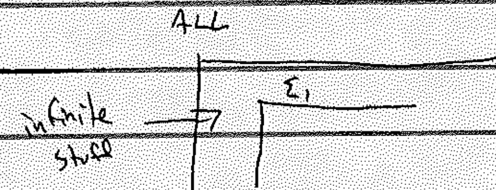
TM

$\approx$ ← by encoding 7-tuple

R $\qquad$ N^k for some k

$\not\approx$ $\qquad$ $\approx$

N



$ALL = P(\Sigma^*)$

$\Sigma^* = \varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 100, 101, \ldots$

$\Sigma^* 1 =$ $A \in ALL = \varepsilon, 1, 01, 11, 001, 011, 101$

$A(i) = \underbrace{1}_{0}\ \underbrace{0}_{1}\ \underbrace{1}_{2}\ \underbrace{0}_{3}\ \underbrace{1}_{4}\ \underbrace{0}_{5}\ \underbrace{1}_{6}\ 0\ 1\ 0\ 1\ 0\ 1\ \ldots$