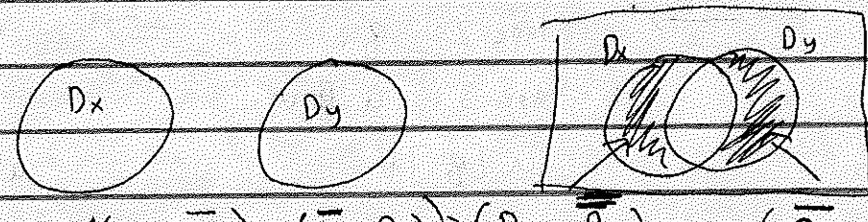
22-1

$A_{DFA} = \{ \langle D, w \rangle \mid D \in DFA,\ w \in L(D) \}$

$E_{DFA} = \{ \langle D \rangle \mid D \in DFA,\ L(D) = \emptyset \} = O(n)$

$EQ_{DFA} = \{ \langle D_x, D_y \rangle \mid D_x, D_y \in DFA,\ L(D_x) = L(D_y) \} = O(n^4)$



$EQ_{DFA}(\langle D_x, D_y \rangle) = E_{DFA}(\langle (D_x \cap \overline{D_y}) \cup (\overline{D_x} \cap D_y) \rangle) \quad (D_x \cap \overline{D_y}) \cup (\overline{D_x} \cap D_y) == \emptyset$

n^4

$LE_{DFA} = \{ \langle D_x, D_y \rangle \mid D_x, D_y \in DFA,\ L(D_x) \subseteq L(D_y) \}$

$= E_{DFA}(\langle (D_x \cap \overline{D_y}) \rangle) = O(n^2)$

A, E, EQ, LE are all "meta-programs" (progs about progs)

$A_{CFG} = \{ \langle G, w \rangle \mid G \in CFG,\ w \in L(G) \}$

① generate all derivations, say yes if w appears $\in \Sigma_1$

② convert G into CNF, then generate all derivs of height $\log_2 |w|$ $\in \Sigma_0$

$CFL \subseteq \Sigma_0$ — $comp(G:CFG)(T:TM) = A_{CFG}\{\langle G, * \rangle\}$

$E_{CFG} = \{ \langle G \rangle \mid G \in CFG,\ L(G) = \emptyset \}$ = no deriv (trivial)

$R = \emptyset$	$S \to 0 \mid X$	$S \to 0S \mid 1S \mid SS \mid 2X$	only infinite deriv
	$X \to 1$	$X \to 3X \mid S$	(i.e no way to [illegible]

V_{term} = variables that may reach terminals $\quad V_{unknown}$ = unknow

init: $(\emptyset, V)$

step: ~~[illegible]~~ $\quad \in \Sigma_0$

$A \to t \Rightarrow A \in V_{term}$

$S \to \varepsilon \Rightarrow S \in V_{term}$

$A \to BC \Rightarrow B \in V_{term} \wedge C \in V_{term} \Rightarrow A \in V_{term}$

if V_{term} changes, then go again

o.w. $S \in V_{term}$? yes $\Rightarrow$ NO, no $\Rightarrow$ YES

22-2 $A_{TM} = \{ \langle M, w \rangle \mid M \in TM, \; w \in L(M) \}$

tape 0: $\langle M, w \rangle$ — tape 0: $\langle M, w \rangle$

tape 1: $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ — tape 1: δ

tape 2: $[\,]q_0$ $\Longrightarrow^{*}$ tape 2: q_i $\Longrightarrow^{*}$

tape 3: $[\,]w$ — tape 3: $u \, \blacksquare \, v$

$[q_0]\, w$ — $u\,[q_i]\, v$ — $u'\,[q_a]\, v$

U is a TM ~~Where~~ here $L(U) = A_{TM}$ — Turing Omnibus

Is U a decider? — If $w \notin L(M)$, will U say NO

NO. $U \in \Sigma_1$ (may loop) — or may it run forever?

Is $A_{TM} \in \Sigma_0$? — A_{TM} is "The Halting Problem"

Assume YES. H is the machine (decider)

$$H(\langle M, w \rangle) = \begin{cases} \text{accept} & \text{if } M \text{ accepts } w \\ \text{reject} & \text{if } M \text{ doesn't accept } w \end{cases}$$

(rejects or loops)

D = "On input $\langle M \rangle$, where $M \in TM$,

1. Run H on $\langle M, \langle M \rangle \rangle$
2. Output the opposite of H."

Quine = A statement about itself

$$D(\langle M \rangle) = \begin{cases} \text{accept} & \text{if } M \text{ } \cancel{\text{rejects}} \text{ does not accept } \langle M \rangle \\ \text{reject} & \text{if } M \text{ accepts } \langle M \rangle \end{cases}$$

what does $D(\langle D \rangle) = ?$

$$= \begin{cases} \text{accept} & \text{if } D \text{ does not accept } \langle D \rangle \\ \text{reject} & \text{if } D \text{ accepts } \langle D \rangle \end{cases}$$

"I am a liar" "This sentence is false" Liar's Paradox

Either D is incorrect or D runs forever

$\Leftrightarrow$ H is incorrect or H runs forever

then $L(H) \neq A_{TM}$ — then H is not decider

untrustworthy & useless — useless

$\Longrightarrow A_{TM} \notin \Sigma_0$