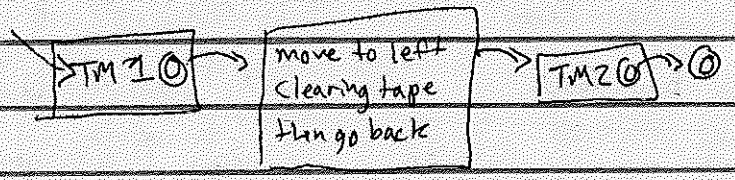


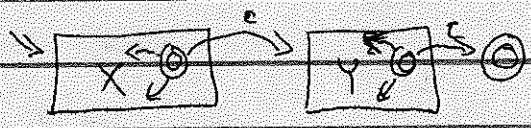
20-1

Spacemacs  
github.com/jeapostrophe/exp/emacs.el

concatenation	•	composition	$[q_0]w \Rightarrow u[q_n]x$
Kleene star	*	$f(w) = x$	
		$g(x) = y$	
$w \in L(TM_1 \cdot TM_2)$		$\Rightarrow fog(w) = y$	
iff $w = x \cdot y$		$f(g(w)) = y$	
$x \in L(TM_1) \wedge y \in L(TM_2)$			



NFAs for  $X \cdot Y$



$X = 0^*$     $Y = 01^*$   
 $X \cdot Y = 0^*01^*$

$x(x_{curr}, c) = (x_{next}, acc?)$   
 if (acc?)  
 fork use y  
 end  
 continue w/x

$x$ : state  $x$  char  $\rightarrow$  state  $x$  bool

$TM_L = ww^R$     $TM_R = 0^n 1^n 0^n 1^n$

$[TM_L, q_0] ww^R$  (hidden:  $0^n 1^n 0^n 1^n$ )

$\Rightarrow^* \dots [q_n] \dots$

$[R, q_0]$  hidden

$\Rightarrow^* \dots [q_n] \dots$

we need to divide string first!

$TM_L \cdot TM_R(w) :=$

consider all divisions of  $w$  into  $x$  &  $y$  s.t.  $w = x \cdot y$

choose  $\perp$  (nondet)

then run  $TM_L(x)$     $\Sigma_0 \cdot \Sigma_0 \in \Sigma_0$

run  $TM_R(y)$     $\Sigma_0 \cdot \Sigma_1 \in \Sigma_1$

if both say yes  $\Rightarrow$  yes    $\Sigma_1 \cdot \Sigma_1 \in \Sigma_1$

if either say no  $\Rightarrow$  no

20-2)  $w \in A^*$

iff  $w = x_0 \cdot \dots \cdot x_n$

where  $x_i \in A$

Kleene (TM  $\Sigma$ ) :=

consider all  $n$  divisions of  $w$  ( $\text{len } k = |w|$ )

consider all ways of divide  $w$  into  $n$  blocks

test each one on  $\text{TM}_\Sigma$

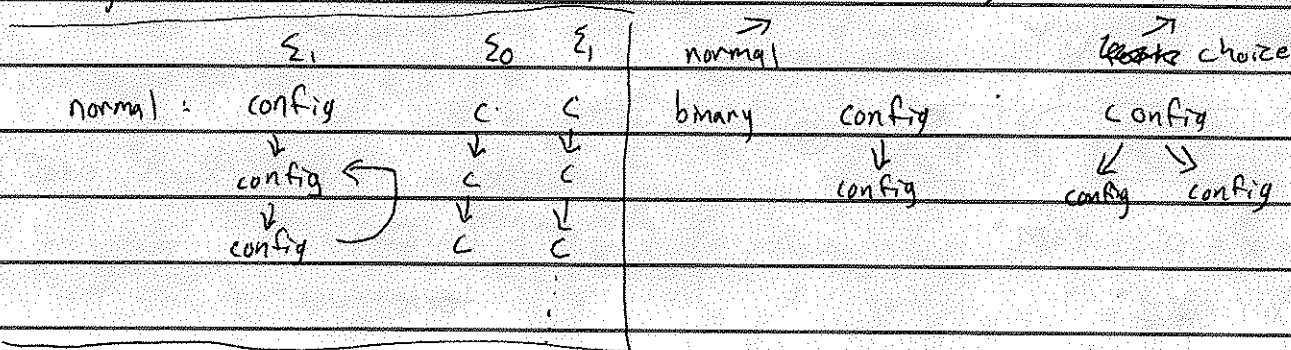
$\Sigma_0^* \in \Sigma_0 \quad \Sigma_1^* \in \Sigma_1$

want: non-deterministic Turing machines

original:  $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$

non-det:  $\delta: Q \times \Gamma \rightarrow P(Q \times \Gamma \times \{L, R\})$

binary <sup>choice</sup> fork:  $\delta: Q \times \Gamma \rightarrow (Q \times \Gamma \times \{L, R\}) + (Q \times Q)$



interpretations of non-det:

- oracle: know the correct path

- fork: parallelism w/ concurrent machines

$$\delta(q_i, a) = (q_j, q_k)$$

$$u[q_i]av \Rightarrow u[q_j]av$$

$$\text{or } u[q_i]av \Rightarrow v[q_k]av$$

$$\delta(q_i, a) = (q_j, b, R)$$

$$\delta(q_i, a) = (q_j, q_k)$$

$$\{u[q_i]av\} \cup X$$

$$\{u[q_i]av\} \cup X \Rightarrow^* \{u[q_j]av, u[q_k]av\} \cup X$$

$$\Rightarrow^* \{u b [q_j] v\} \cup X$$

②

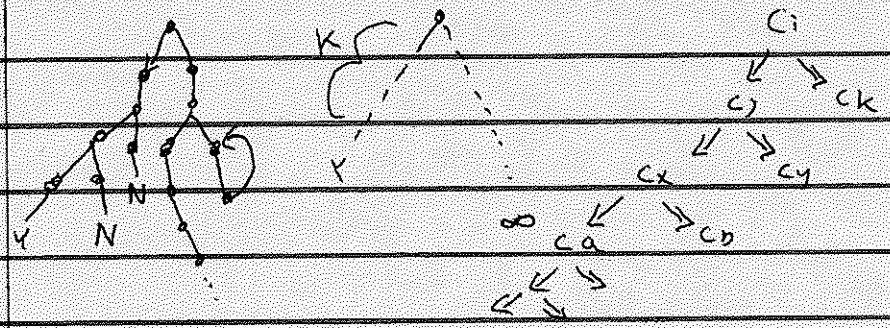
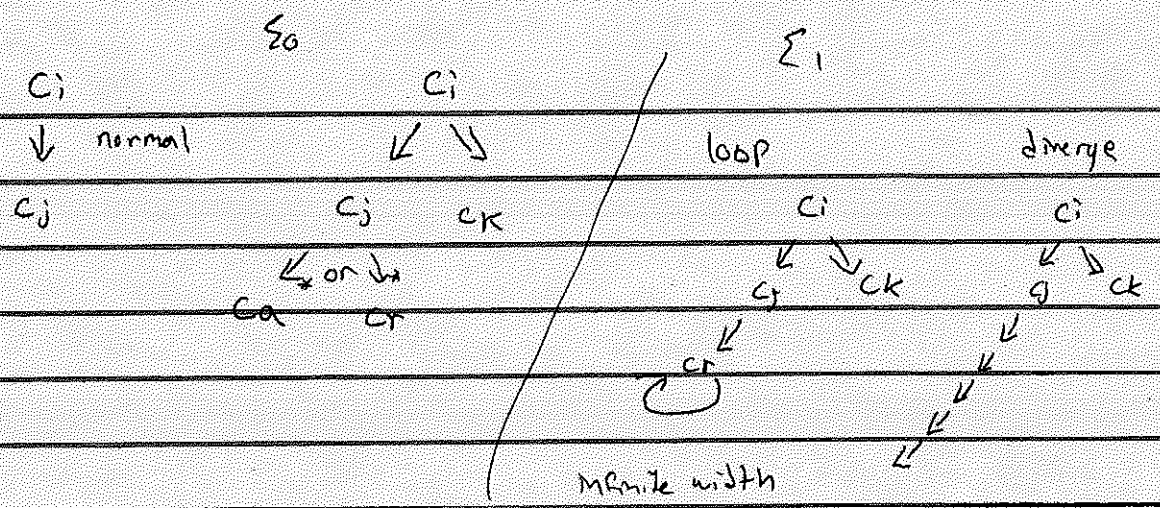
$$\{u_0 [q_{i_0}] v_0, \dots, u_n [q_{i_n}] v_n\} \Rightarrow$$

$$\# u_0 q_{i_0} v_0 \# \dots \# u_n q_{i_n} v_n \#$$

②

$$\# u_0 q_{i_0} v_0 \# u_0 q_{i_0} v_0 \# \dots \# u_n q_{i_n} v_n \#$$

back-tracking : try 1 thing, then try something else



find  $\gamma = \text{accept}$   
 know no  $\gamma$ s = reject  
 $\Sigma_0$  can't  $\Sigma_1$

abstract "Yarn of length k"  $\rightarrow$  tape 1: w (original input)  
 attempt n tape 2: current simulation  
 tape 3: path from  $\{L, S, R\}^*$

