2-1

$\Sigma = \{a, b, ☺, ⊗\}$

$w = a ☺ b ⊗ aabbab$

A "language" is a set of strings

$\{ab, ba, aa, bb\} = X$

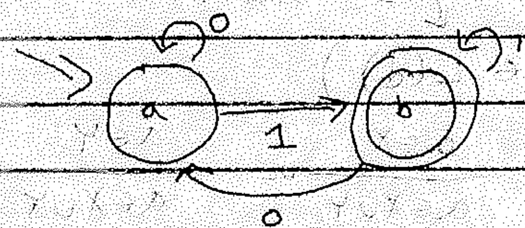
$\Sigma = \{0, 1\}$ Odds = "All strings that rep odd numbers"

= "All strings ending in a 1."

000 ∈ X

010 ∈ X

011 ∈ ✓



1+2 = 7

∈ DFAs = deterministic finite automata

○ := states (a, b)

one state has an arrow from nowhere

→○ := the start state (a)

some states have two circles

◎ ◎ := accept states (b)

states are connected w/ edges

$x \xrightarrow{c} y$:= the transitions

rules!

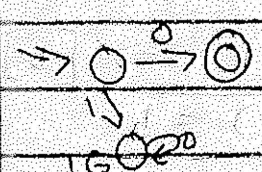
(a, 0, a) (a, 1, b) (b, 1, b) (b, 0, a)

- transitions are function-like] $(\forall x, \forall c, \forall y_2\, \forall y_1\; (x, c, y_1) \in Trans$

$\wedge\, (x, c, y_2) \in Trans$

$\Rightarrow y_1 = y_2)$

$a \in (X \cup Y) \Leftrightarrow a \in X$ or $a \in Y$



Finite sets = FIN

Sets defined by ~~DFA~~ DFAs

= Regular sets

= REG

⋮

All sets = ALL

2-2 / A DFA d is $(Q, \Sigma, q_0, \delta, F)$

Q = the states, Σ = the alphabet, q_0 = the start state, δ = the transition function, F = the accepting states

Q : set $\in$ Finite Sets

Σ : alphabet := set that's finite

$q_0 \in Q$

$\delta : Q \times \Sigma \Rightarrow Q \qquad \delta(x, c) = y \qquad (x) \xrightarrow{c} (y)$

$F \subseteq Q$

$\dfrac{\text{premises}}{\text{conclusion}}$ name

$L : \text{DFA} \Rightarrow \text{Set}$

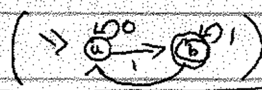
A string $w \in$ a DFA d $\quad (w \in L(d))$

$a \in X \cup Y$ iff $a \in X$ or $a \in Y \qquad \dfrac{a \in X}{a \in X \cup Y}\,\text{UL} \qquad \dfrac{a \in Y}{a \in X \cup Y}\,\text{UR}$

$$\dfrac{[q_0]\,w \Rightarrow^* [q_f]\,\varepsilon \qquad q_f \in F}{w \in L(d)}\ (1) \qquad \underbrace{[q_i]}_{Q}\,\underbrace{w}_{\text{str of } \Sigma} \Rightarrow^* \underbrace{[q_j]}_{Q}\,\underbrace{w'}_{\text{str of } \Sigma}$$

"q_i runs on input w to q_j with input w'"

$$\dfrac{}{[q_i]\,\varepsilon \Rightarrow^* [q_i]\,\varepsilon}\ (2) \qquad \dfrac{\delta(q_i, a) = q_j}{[q_i]\,aw \Rightarrow^* [q_j]\,w}\ (3)\ \ a \in \Sigma \qquad \dfrac{[q_i]\,w \Rightarrow^* [q_j]\,w' \qquad [q_j]\,w' \Rightarrow^* [q_k]\,w''}{[q_i]\,w \Rightarrow^* [q_k]\,w''}\ (4)$$

$011 \in L($  $)$ (1)

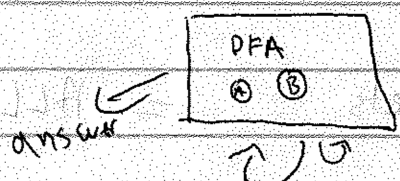
$[a]\,011 \Rightarrow^* [b]\,\varepsilon$ (4)

$[a]\,011 \Rightarrow^* [a]\,11$ (3) $\qquad [a]\,11 \Rightarrow^* [b]\,\varepsilon$ (4)

$\delta(a, 0) = a \qquad [a]\,11 \Rightarrow^* [b]\,1$ (3) $\qquad [b]\,1 \Rightarrow^* [b]\,\varepsilon$ (3)

$\delta(a, 1) = b \qquad \delta(b, 1) = b$

$[a]\,011 \Rightarrow [a]\,11 \Rightarrow [b]\,1 \Rightarrow [b]\,\varepsilon$



$\Leftarrow$ input =

formula	ex1	ex2
$\lvert Q \rvert = n$	2	32
rt = $\lvert in \rvert$	$\lvert in \rvert$	$\lvert in \rvert$
mem = $\log_2 n$ = $\lg n$	1 bit	5 bits

2-3)

FIN = finite sets

REG = regular sets (i.e. those defined by DFAs)

ALL = all the sets

✓ FIN $\subsetneq$ ALL

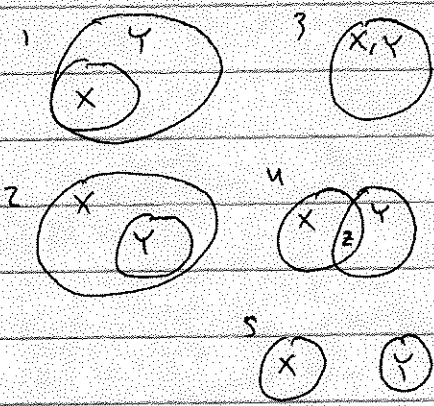
✓ REG $\subseteq$ ALL

REG = ALL ?

REG $\neq$ ALL ?

✓ FIN $\neq$ ~~ALL~~ REG

FIN $\subsetneq$ ~~ALL~~ REG ?



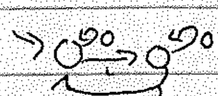
$X \not\subset Y$:=

1. $X \subset Y$
2. $Y \subset X$
3. $X = Y$
4. $X \cap Y \neq \emptyset$
5. $X \cap Y = \emptyset$

$\forall F \in FIN, \ \exists \underbrace{R \in REG}_{\text{means } \exists \text{ some DFA}}, \ F = R$

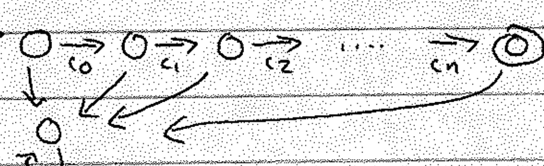
$\forall F \in FIN, \ \exists d \in DFA, \ L(d) = F$

consider $F = \emptyset$, $d =$ 

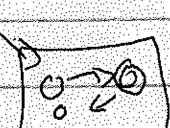
$d =$ 

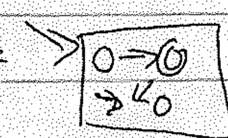
$F = \{w_0\}$

$w_0 = c_0 c_1 c_2 \ldots c_n$

$d =$ 

$F = F_x \cup F_y$

$F_x = d_x =$ 

$F_y = d'_y =$ 

$(Q_x, \Sigma, q_{0x}, \delta_x, F_x)$ $\qquad$ $(Q_y, \Sigma, q_{0y}, \delta_y, F_y)$

$d_{x \cup y} = Q = Q_x \times Q_y$

$\Sigma = \Sigma$

$q_0 = (q_{0x}, q_{0y})$

$\delta((q_{ix}, q_{iy}), a) = (\delta_x(q_{ix}, a), \delta_y(q_{iy}, a))$

$F = F_x \times Q_y \cup Q_x \times F_y$