8-1

TM = DFA w/ infinite tape r/w

DFA $\delta: Q \times \Sigma \to Q$

NFA $\delta: Q \times \Sigma_\varepsilon \to P(Q)$

PDA $\delta: Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \to P(Q \times \Gamma_\varepsilon)$

TM $\delta: \underbrace{Q}_{Q - \{q_a, q_r\}} \times \Gamma \to \underbrace{Q}_{Q - \{q_h\}} \times \Gamma \times \{L, R\}$

distinguished accept

$w \in L(M)$ iff $[q_0]w \Rightarrow^* u[q_a]v$

computable fun (variant of TM) — distinguished halt

$f(w) = v$ iff $[q_0]w \Rightarrow^* u[q_h]v$

$[q_0]w \Rightarrow^*$ $u_0[q_a]v_0$ accepted

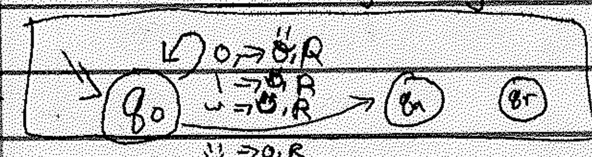
$u_1[q_a]v_1$ also accepted (diff machine)

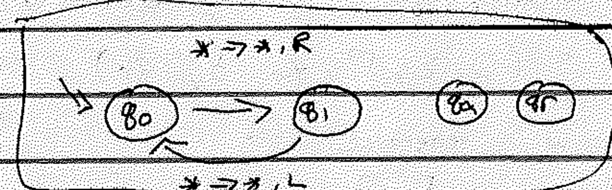
$u_2[q_r]v_2$ rejected

$\forall^{q_i} u_3, v_3.\ [q_0]w \Rightarrow^* u_3[q_i]v_3 \Rightarrow \exists u_4, v_4, q_j$ $u_4[q_j]v_4$ s.t. $q_j \neq q_a, q_r$

runs infinitely long

$\Gamma =$ $0, 1, \sqcup, \ddot{\sqcup}$





$[q_0]011 \Rightarrow \ddot{\sqcup}[q_0]11 \Rightarrow \ddot{\sqcup}\ddot{\sqcup}[q_0]1$

$\ddot{\sqcup}\ddot{\sqcup}\ddot{\sqcup}[q_0]\sqcup \Rightarrow \ddot{\sqcup}\ddot{\sqcup}\ddot{\sqcup}\ddot{\sqcup}[q_0]$

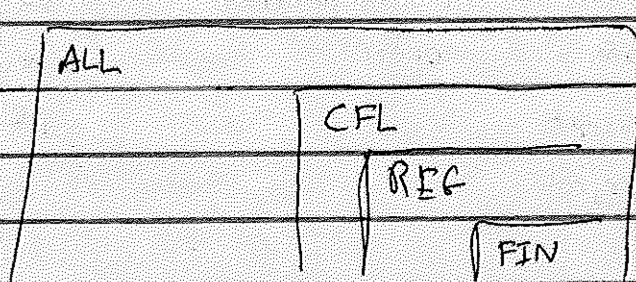
i steps $\ddot{\sqcup}^i[q_0]$

diverge

$[q_0]011 \Rightarrow 0[q_0]11 \Rightarrow$

$[q_0]011 \longrightarrow$

loop



18-2

Turing-decidable languages Σ_0

Turing-recognizable languages Σ_1

A decider is a TM that never runs forever

decider (t) := $\forall w \in \Sigma^*.\ [q_0]\,w \Rightarrow^* u\,[q_a]\,v$
or $[q_0]\,w \Rightarrow^* u'\,[q_r]\,v$

A recognizer is just not a decider (or all TMs)

recognizer (t) := $\neg$ decider (t)

$A \in \Sigma_0$ iff $\exists t \in TM.$ $L(t) = A \wedge$ decider(t)

$A \in REG$ iff $\exists d \in DFA.\ L(d) = A$

$A \in \Sigma_1$

iff $(\exists t \in TM.\ L(t) = A)$

$\Sigma_0 \subseteq \Sigma_1$

Qs: $\Sigma_0 = \Sigma_1$? $\Sigma_1 = ALL$? What about REG, CFL?

Enumerator

not restricted

$Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$

$= (Q, \Sigma, \Gamma, q_0, \delta, q_P)$

w is generated iff

$[q_0] \Rightarrow^* u\,[q_P]\,w$

$u \in \Gamma^*$ $w \in \Sigma^*$

q_P is NOT an end state

DFA	REX
PDA	CFG
TM	enum
machines	PLs
$w \to B$	$\to A$

CFG $A \to \beta$

$\in V$ $\in (V \cup \Sigma)^*$

$\alpha \to \beta$

$\in (V \cup \Sigma)^*$

when is an enumerator "decidable"?

$\Downarrow$ prints in partial order

Decide (w) :=

turn on enum

if == w, accept

if > |w|, reject

turn an enum into recognizer

Recognize (w) :=

turn on enum

check if each string == w

if so, Accept

18-3

$REG \in \Sigma_0$ "Every regular language is decidable"

$\forall A$ $A \in REG \rightarrow A \in \Sigma_0$

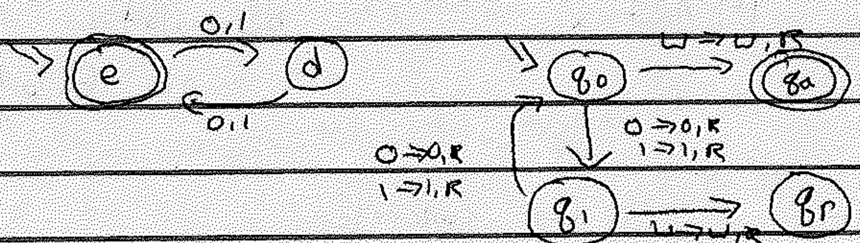
$\forall A$

$(\exists d \in DFA.\ L(d) = A) \rightarrow (\exists t \in TM.\ L(t) = A \wedge \text{decider}(t))$

compiler : DFA $\rightarrow$ TM

in : $(Q, \Sigma, q_0, \delta : Q \times \Sigma \rightarrow Q, F \subseteq Q)$

out: $(Q', \Sigma, \Gamma, q_0', \delta' : Q \times \Gamma \rightarrow Q' \times \Gamma \times \{L, R\}, q_a, q_r)$



$Q' = Q \cup \{q_a, q_r\}$ $\quad \Gamma = \{\sqcup\}$

$q_0' = q_0$ $\quad \delta'(q_i, a) = (\delta(q_i, a), \sqcup, R)$

$\delta'(q_i, \sqcup) = \left(\begin{cases} q_a & \text{if } q_i \in F \\ q_r & \text{o.w} \end{cases}, \sqcup, R \right)$

DFA: $[e]0101 \rightarrow [d]101 \rightarrow [e]01 \rightarrow [d]1 \rightarrow [e] \rightarrow \checkmark$

TM: same $\quad [e] \rightarrow [q_a] \checkmark$

18-4 Is Σ_0 or Σ_1 closed under : $\cup, \cap, \circ, *, R, \overline{\ }$

union : combine TMs X and Y into Z

s.t. $w \in Z$ iff $w \in X$ or $w \in Y$

$$Q_Z = Q_X \times Q_Y$$

$$\delta_Z((q_X, q_Y), a) = ((q_X', q_Y'), ???, ???)$$

where $(q_X', b_X, d_X) = \delta(q_X, a)$

$(q_Y', b_Y, d_Y) = \delta_Y(q_Y, a)$

$$\text{DFA} \xrightarrow{\text{trivial}} \text{DFA} + x \qquad \text{DFA} + x \xrightarrow{\text{compiler}} \text{DFA}$$

bisimulation

aka Galois connection

A = abstract C = concrete

$f : A \to C$ g is not necessarily f^{-1}

$g : C \to A$

$m_{a1} : \vec{A} \to A$ $m_{c1} : \vec{C} \to C$

$f(m_{a1}(a)) = m_{c1}(f(a))$ $a = g(c)$

$g(m_{c1}(c)) = m_{a1}(g(c))$ $\Rightarrow a = g(f(a))$

$g(f(a)) = a$ ~~f(g(c)) = c~~

$A = \{+, -, 0\}$ $C = \mathbb{N}$

$f(+) = 1$ $f(-) = -1$ $f(0) = 0$ $x_a(+,+) \to +$

$g(-n) = -$ $g(+n) = +$ $g(0) = 0$ $(+,-) \to -$

$x_c = x$

A = queues C = linked list

C' = heaps

C'' = binomial heaps