

24-1

$$T_1 = T_1 \rightarrow T_2$$

$$T = B \mid T \rightarrow T \mid T + T \mid T \times T \mid \forall A. T \mid A$$

$$\text{List of Num s} = \text{NULL} + (\text{Num} \times \text{List of Num s})$$

$$T = \dots \mid \text{fix } T \quad (\text{no modifications to } M \text{ or } V)$$

$$\text{List } N = \text{fix } (\forall A. \text{NULL} + (\text{num} \times A))$$

$$\text{fix } (\forall A. T) = \mu A. T$$

conversion relation $T \leftrightarrow T$

$$\text{old: } \Gamma \vdash M : T_1 \rightarrow T_2 \quad \Gamma \vdash N : T_1 \quad \Gamma \vdash (M \ N) : T_2$$

$$\text{new: } \Gamma \vdash M : T_1 \quad \Gamma \vdash N : T_2 \quad T_1 \leftrightarrow (T_2 \rightarrow T_3) \quad \Gamma \vdash (M \ N) : T_3$$

$$B \leftrightarrow B \quad T_1 \leftrightarrow T_3 \quad T_2 \leftrightarrow T_4 \quad (T_1 \rightarrow T_2) \leftrightarrow (T_3 \rightarrow T_4)$$

(sym)

$$T_1 [A \leftarrow \mu A. T_1] \leftrightarrow T_2 \quad [1; 2; 3] = \text{incr}(1, \text{incr}(2, \text{incr}(3, \text{nil})))$$
$$\mu A. T_1 \leftrightarrow T_2 \quad (\text{nil} + (\text{num}, \text{nil} + (\text{num}, \text{nil} + \text{num}(\dots)))$$

$\perp = \text{nil}$
~~0~~

$$\text{length } (x : \text{List num}) :=$$

$$\text{case } x \text{ with } \Rightarrow 0 \ ; \ p \Rightarrow 1 + \text{length } (\text{snd } p)$$

$$(\mu A. \text{nil} + (\text{num} \times A)) \leftrightarrow \perp + (\text{num}, 1 + (\text{num}, 1 + (\text{num}, 1 + (\text{num}, 1))))$$

$$T_1 [A \leftarrow C] \leftrightarrow T_2 [B \leftarrow C] \text{ where } C \text{ is fresh}$$

$$\mu A. T_1 \leftrightarrow \mu B. T_2$$

equi-recursive type system

equality between type

24-2/ $T := \dots \mid \mu A.T$

$M := \dots \mid \text{unfold } M \mid \text{fold } M$

$V := \dots \mid \text{fold } V$

$E := \dots \mid \text{unfold } E \mid \text{fold } E$

$E[\text{unfold}(\text{fold } V)] \Rightarrow E[V]$

$\Gamma \vdash M : T [A \leftarrow \mu A.T]$

$\Gamma \vdash \text{fold } M : \mu A.T$

$\Gamma \vdash M : \mu A.T$

$\Gamma \vdash \text{unfold } M : T [A \leftarrow \mu A.T]$

constructors always call fold

destructors always call unfold

$\text{Listnum} = \mu A. 1 + (\text{num} \times A)$

$\text{null} := \text{fold}(\text{in } \text{false}) : \text{Listnum}$

$\text{cons} := (\lambda x:\text{num}. (\lambda r:\text{Listnum}. \text{fold}(\text{in } (\text{pair } x \ r))))$

$: \text{num} \rightarrow \text{Listnum} \rightarrow \text{Listnum}$

$\text{fst} := (\lambda l:\text{Listnum}.$

case (unfold l)

- \Rightarrow error

p \Rightarrow fst p

24-3/

$T ::= B \mid T \rightarrow T \mid T + T \mid T \times T$
 $\mid A \mid \forall A. T \mid \text{let } x T$

$T ::= \dots \mid 0 \mid 1$ $\Gamma + t : 1$

~~$M ::= \dots \mid \dots \mid ++ / () / \text{unit}$~~

$\forall m. \Gamma \vdash M = 0$

$\forall x. 0 + x = x$
 ~~$\text{int } (M)$~~ when $M = 0$
 $\text{imm } x \Rightarrow x$

$f : \rightarrow 0$ $m : \rightarrow x$ $x ::= \text{if } c \text{ in } f \text{ in } m$ $\text{case } x. \text{ with}$ $_ \Rightarrow \text{for } f$ $_ \Rightarrow \text{for } m$	$f = 0$ $m \Rightarrow x$ $x ::= \text{if } c \text{ then } f \text{ else } m$ $\text{for } m$
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$x + 0 = x$

$1 \times a = a = a \times 1$

$0 \times a = 0$

$(a \times b) \times c = a \times (b \times c)$

$\nearrow \text{fst} \quad \nearrow \text{snd} \quad \nearrow \text{fst} \quad \nearrow \text{fst} \quad \nearrow \text{snd}$

$\text{Bool} = 1 + 1$

$\text{Nat} = \mu A. 1 + A$

$0 = \text{fold}(\text{inl } \text{tt})$

$1 = \text{fold}(\text{inr}(\text{inl } \text{tt}))$

$2 = \text{fold}(\text{inr}(\text{inr}(\text{inl } \text{tt})))$

$d_x (7x^2 + 3x + 1) = 14x + 3$

$L\langle A \rangle = \forall A. \mu L. 1 + (A \times L)$

$T\langle A \rangle = \forall A. \mu T. A + (T \times T)$

(algebraic datatypes)

$\text{ListNum} = 1 + (\text{Num} \times \text{ListNum})$

$\partial_n \text{ListNum} = \partial_n (1 + (\text{Num} \times \text{ListNum}))$

$= \partial_n 1 + \partial_n (\text{Num} \times \text{ListNum})$

$= 0 + \partial_n (\text{Num} \times \text{ListNum})$

$= \partial_n (\text{Num} \times \text{ListNum})$

$= \partial_n \text{Num} \times \text{ListNum} + \text{Num} \times \partial_n \text{ListNum}$

$= 1 \times \text{ListNum} + \text{Num} \times \partial_n \text{ListNum}$

$\partial_n \text{ListNum} = \text{ListNum} + \text{Num} \times \partial_n \text{ListNum}$

$= \mu L. \text{ListNum} + \text{Num} \times L$

$\partial_x (A + B) = \partial_x A + \partial_x B$

$d_x k = 0$

$d_x (A \cdot B) = (d_x A) \cdot B + A \cdot (d_x B)$

$d_x x = 1$

24-4

NL = empty | cons num NL

D = boring NL | interesting num D

D is called a Zipper

|| "This is my file" \Rightarrow boring "This is my file"

"This" | "is my file" \Rightarrow *

(S, i, h, T) | (boring "is my file")

move right (before, (next: after)) = (next: before, after)

left ((prev: before), after) = (before, prev: after)

insert k (before, after) = (~~before~~ k: before, after)

gap list