7-1)

Evaluation ($eval_v^s$)

1. Check if M is a value? If so, stop
2. Separate into the eval ctx $\underline{E}$ and redex M — reducible expression
3. Apply β_v or Δ on M to get M'
4. Produce $E[M']$

Standard Reduction:

Program M
while (M isn't V)
  $E[N] = M$
  $N' = v(N)$
  $M = E(N')$

~~Program~~ EvalCtx E, Program M
while
  $M' = v(M)$
  update E, M
  $E'[N] = E[M']$

CC-machine
- evaluation context
- control string, M

$\longmapsto_{cc} : \langle M, E \rangle \rightarrow \langle M, E \rangle$

[1] $\langle (M\ N), E \rangle \longmapsto_{cc} \langle M, E[([\,]\ N)] \rangle$
  $M \notin V$

[2] $\langle (V\ N), E \rangle \longmapsto_{cc} \langle N, E[(V\ [\,])] \rangle$
  $N \notin V$

[3] $\langle ((\lambda X.M)\ V), E \rangle \longmapsto_{cc} \langle M[X \leftarrow V], E \rangle$

[4] $\langle V, E[([\,]\ N)] \rangle \longmapsto_{cc} \langle (V\ N), E \rangle$

[5] $\langle V, E[(U\ [\,])] \rangle \longmapsto_{cc} \langle (U\ V), E \rangle$

$M \notin V$ $\langle (o^n\ V \ldots\ M\ N \ldots), E \rangle \longmapsto_{cc} \langle M, E[(o^n\ V \ldots\ [\,]\ N \ldots)] \rangle$

$\langle (o^n\ b \ldots), E \rangle \longmapsto_{cc} \langle \delta(o^n, b \ldots), E \rangle$

$\langle V, E[(o^n\ U \ldots\ [\,]\ N \ldots)] \rangle \longmapsto \langle (o^n\ U \ldots\ V\ N \ldots), E \rangle$

7-2

$$\text{S.r.}\quad \overbrace{1\ 2\ 1}^{\text{parsing}}\ 3\ \overbrace{4\ 5\ 4}\ \underbrace{1\ 2\ 1}\ 3\ \overbrace{4\ 5\ 4}^{\text{filling}}$$

CC 1 2 1 3 3 4 5 4

cc 1 1 3 2 3 3 5 4 4

$eval_{cc}(M)$ = let ~~V~~ V be $\langle M, [\,]\rangle \longmapsto\!\!\!\to_{cc} \langle V, [\,]\rangle$
if V is a b, then ret b
V is a λ, then ret 'fun

Theorem: $eval_{cc} = eval_v^s$
$\Leftrightarrow \forall M, V.\quad M \longmapsto\!\!\!\to_v V$ iff $\langle M, [\,]\rangle \longmapsto\!\!\!\to_{cc} \langle V, [\,]\rangle$
generalize $\forall M, E, V.\quad E[M] \longmapsto\!\!\!\to_v V$ iff $\langle M, E\rangle \longmapsto\!\!\!\to_{cc} \langle V, [\,]\rangle$

Lemma: If $M = E'[L]$ and $L\ v\ L'$ then
$\langle M, E\rangle \longmapsto\!\!\!\to_{cc} \langle L, E[E']\rangle$

SCC - simplified CC-machine
$\longmapsto_{scc} : \langle M, E\rangle \longrightarrow \langle M, E\rangle$

[1] $\langle (M\ N), E\rangle \longmapsto_{scc} \langle M, E[([\,]\ N)]\rangle$
[2] $\langle V, E[([\,]\ N)]\rangle \longmapsto_{scc} \langle N, E[(V\ [\,])]\rangle$
[3] $\langle V, E[(U\ [\,])]\rangle \longmapsto_{scc} \langle M[X \leftarrow V], E\rangle$
if $U = \lambda X. M$

[4] $\langle (o^n\ M\ N \ldots), E\rangle \longmapsto_{scc} \langle M, E[(o^n\ [\,]\ N \ldots)]\rangle$
[5] $\langle V, E[(o^n\ U \ldots\ [\,]\ M\ N \ldots)]\rangle \longmapsto_{scc} \langle M, E[(o^n\ U \ldots\ V\ [\,]\ N \ldots)]\rangle$
[6] $\langle b_n, E[(o^n\ b_1 \ldots b_{n-1}\ [\,])]\rangle \longmapsto_{scc} \langle \delta(o^n, b_1, \ldots, b_n), E\rangle$

7-3 $\langle (+\ ((\lambda x.x)\ 3)\ ((\lambda y.y)\ 4)),\ [\,]\rangle$

$4 \longmapsto_{scc} \langle ((\lambda x.x)\ 3),\ (+\ [\,]\ ((\lambda y.y)\ 4))\rangle$

$1 \longmapsto_{scc} \langle (\lambda x.x),\ (+\ ([\,]\ 3)\ ((\lambda y.y)\ 4))\rangle$

$2 \longmapsto_{scc} \langle 3,\ (+\ ((\lambda x.x)\ [\,])\ ((\lambda y.y)\ 4))\rangle$

$3 \longmapsto_{scc} \langle x[x \leftarrow 3] = 3,\ (+\ [\,]\ ((\lambda y.y)\ 4))\rangle$

$5 \longmapsto_{scc} \langle ((\lambda y.y)\ 4),\ (+\ 3\ [\,])\rangle$

$1 \longmapsto_{scc} \langle (\lambda y.y),\ (+\ 3\ ([\,]\ 4))\rangle$

$2 \longmapsto_{scc} \langle 4,\ (+\ 3\ ((\lambda y.y)\ [\,]))\rangle$

$3 \longmapsto_{scc} \langle y[y \leftarrow 4] = 4,\ (+\ 3\ [\,]\rangle$

$6 \longmapsto_{scc} \langle \delta(+, 3, 4) = 7,\ [\,]\rangle$

$\langle 7,\ [\,]\rangle$

suppose

$(+\ (+\ ((\lambda x.x)\ 3)\ ((\lambda y.y)\ 4))\ 7)$

$\langle 7,\ (+\ [\,]\ 7)\rangle$

$\langle 7,\ (+\ 7\ [\,])\rangle$

$\langle 14,\ [\,]\rangle$