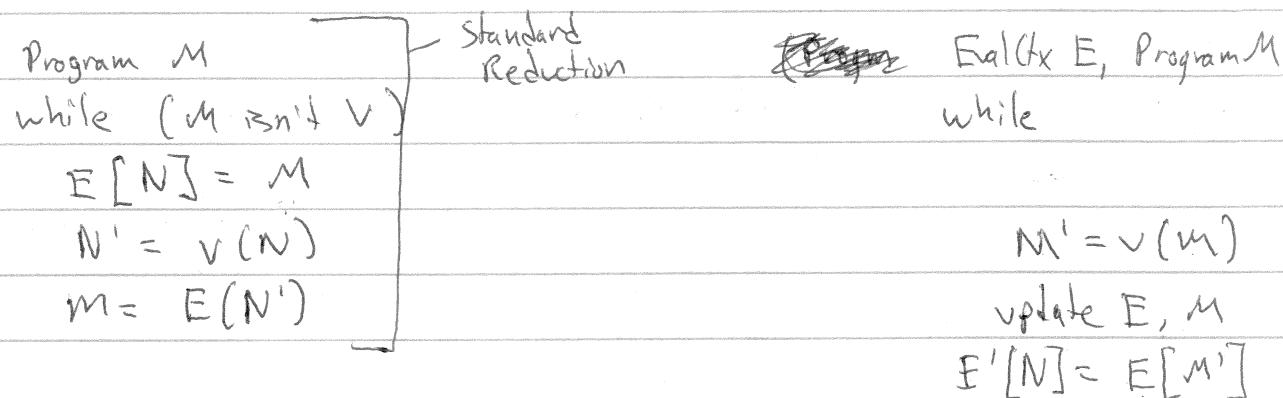


7-1)

Evaluation (eval_v^s)

- ↓
1. Check if m is a value? If so, stop / reducible expression
 2. Separate into the eval ctx E and redex M
 3. Apply β_v or Δ on M to get M'
 4. Produce $E[M']$



evaluation context
 (C-Machine)
 ↓ control string, M

$$\xrightarrow{\text{cc}} : \langle M, E \rangle \rightarrow \langle M, E \rangle$$

$\boxed{E1} \quad \langle (M N), E \rangle \xrightarrow{\text{cc}} \langle M, E[(\boxed{E} N)] \rangle$
 $M \notin V$

$$\boxed{E2} \quad \langle (V N), E \rangle \xrightarrow{\text{cc}} \langle N, E[(V \boxed{E})] \rangle$$

 $N \notin V$

$$\boxed{E3} \quad \langle (\lambda x.M) V, E \rangle \xrightarrow{\text{cc}} \langle M[x \leftarrow V], E \rangle$$

$$\boxed{E4} \quad \langle V, E[(\boxed{E} N)] \rangle \xrightarrow{\text{cc}} \langle (V N), E \rangle$$

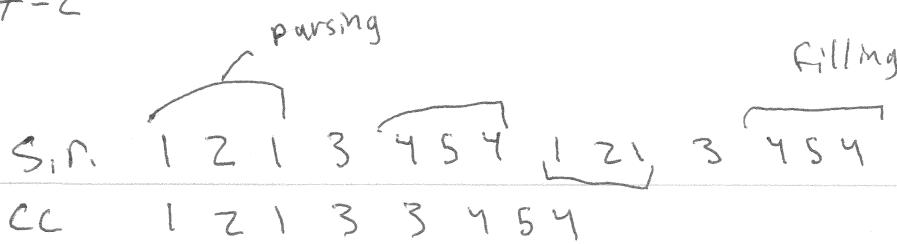
$$\boxed{E5} \quad \langle V, E[(U \boxed{E})] \rangle \xrightarrow{\text{cc}} \langle (U V), E \rangle$$

$$m \in V \quad \langle (o^n V \dots M N \dots), E \rangle \xrightarrow{\text{cc}} \langle M, E[(o^n V \dots \boxed{E} N \dots)] \rangle =$$

$$\langle (o^n b \dots), E \rangle \xrightarrow{\text{cc}} \langle S(o^n, b \dots), E \rangle$$

$$\langle V, F \Gamma (o^n U \dots \boxed{E} N \dots) \Gamma \rangle \xrightarrow{\text{cc}} \langle (o^n U \dots V N \dots), E \rangle$$

7-2



$\text{eval}_{\text{CC}}(m) = \text{let } \cancel{\text{eval}_V} V \text{ be } \langle M, [] \rangle \mapsto_{\text{CC}} \langle V, [] \rangle$
if V is ab , then ret b
 V is a λ , then ret $'$ fun

Theorem: $\text{eval}_{\text{CC}} = \text{eval}_V^S$

$\Leftrightarrow \forall m, v. \quad m \mapsto_v V \text{ iff } \langle M, [] \rangle \mapsto_{\text{CC}} \langle V, [] \rangle$
generalize $\forall m, E, v. \quad E[m] \mapsto_v V \text{ iff } \langle M, E \rangle \mapsto_{\text{CC}} \langle V, [] \rangle$

Lemma: If $m = E[L]$ and $L \vee L'$ then
 $\langle M, E \rangle \mapsto_{\text{CC}} \langle L, E[E'] \rangle$

SCC - simplified CC-machine

$\mapsto_{\text{SCC}} : \langle M, E \rangle \rightarrow \langle M, E \rangle$

3) $\langle (M N), E \rangle \mapsto_{\text{SCC}} \langle M, E[(\lambda [] N)] \rangle$

3) $\langle V, E[(\lambda [] N)] \rangle \mapsto_{\text{SCC}} \langle N, E[(\lambda [] V)] \rangle$

3) $\langle V, E[(\lambda [])] \rangle \mapsto_{\text{SCC}} \langle M[x \leftarrow V], E \rangle$
if $\lambda = \lambda x, M$

3) $\langle (\lambda^n M N \dots), E \rangle \mapsto_{\text{SCC}} \langle M, E[(\lambda^n [] N \dots)] \rangle$

3) $\langle V, E[(\lambda^n u \dots [] M N \dots)] \rangle \mapsto_{\text{SCC}} \langle M, E[(\lambda^n u \dots v [] N \dots)] \rangle$

3) $\langle b_n, E[(\lambda^n b_1 \dots b_{n-1} [])] \rangle \mapsto_{\text{SCC}} \langle \delta(\lambda^n, b_1, \dots, b_n), E \rangle$

- 7-3 $\langle (+ ((\lambda_{x,x}) 3) ((\lambda_{y,y}) 4)), [] \rangle$
- $4 \mapsto_{\text{scc}} \langle ((\lambda_{x,x}) 3), (+ [] ((\lambda_{y,y}) 4)) \rangle$
- $1 \mapsto_{\text{scc}} \langle (\lambda_{x,x}), (+ ([] 3) ((\lambda_{y,y}) 4)) \rangle$
- $2 \mapsto_{\text{scc}} \langle 3, (+ ((\lambda_{x,x}) []) ((\lambda_{y,y}) 4)) \rangle$
- $3 \mapsto_{\text{scc}} \langle x[x \leftarrow 3] = 3, (+ [] ((\lambda_{y,y}) 4)) \rangle$
- $5 \mapsto_{\text{scc}} \langle ((\lambda_{y,y}) 4), (+ 3 []) \rangle$
- $1 \mapsto_{\text{scc}} \langle (\lambda_{y,y}), (+ 3 ([] 4)) \rangle$
- $2 \mapsto_{\text{scc}} \langle 4, (+ 3 ((\lambda_{y,y}) [])) \rangle$
- $3 \mapsto_{\text{scc}} \langle y[y \leftarrow 4] = 4, (+ 3 []) \rangle$
- $6 \mapsto_{\text{scc}} \langle 8(+, 3, 4) = 7, [] \rangle$
- $\langle 7, [] \rangle$
- \swarrow suppose
- $(+ (+ ((\lambda_{x,x}) 3) ((\lambda_{y,y}) 4)) 7)$
- $\langle 7, (+ [] 7) \rangle$
- $\langle 7, (+ 7 []) \rangle$
- $\langle 14, [] \rangle$

