4-1

ISWIM - if you see what I mean

$M, N, L, K =$	X	variables
$\mid$	$(\lambda X. M)$	functions
$\mid$	$(M\ N)$	application (function call)
$\mid$	b	constants from set B
$\mid$	$(o^n\ M_1 \ldots M_n)$	primitive fun calls of arity n from some set O

$B = \{ \text{true}, \text{false} \} \cup \{ \ulcorner n \urcorner \mid n \in \mathbb{Z} \}$ where $\ulcorner n \urcorner$ is the stx for an Integer

$O_1 = \{ \text{not}, \text{negate} \}$

$O_2 = \{ \text{add}, \text{sub}, \text{mul}, \text{div}, \text{expt} \}$

$B = O_0$

FV (free variables) : $M \rightarrow \{ X \ldots \}$

$FV(X) = \{X\}$

$FV(M\ N) = FV(M) \cup FV(N)$

$FV(b) = \emptyset$

$FV(o^n\ M_1 \ldots M_n) = \bigcup_{i=1}^{n} FV(M_i)$

$FV(\lambda X. M) = FV(M) - \{X\}$

$id = \lambda x. x$

substitution

$M[X \leftarrow N]$

$f(\overset{X}{x}) = \underbrace{42 * x + (7+8)}_{M} = 42 * x + 15$

$f(\underbrace{17}_{N}) = 42 * 17 + (7+8)$

$g(x) = \{ \ f(x) = 8 + x ; \ f(x+2) \ \}$

$X[X \leftarrow N] = N$

$Y[X \leftarrow N] = Y$

$(M\ N)[X \leftarrow L] = (M[X \leftarrow L]\ N[X \leftarrow L])$

$(o^n\ M_1 \ldots M_n)[X \leftarrow N] = (o^n\ M_1[X \leftarrow N] \ldots M_n[X \leftarrow N])$

$b[X \leftarrow N] = b$

SHADOWING

$(\lambda X. M)[X \leftarrow N] = (\lambda X. M)$ NOT $(\lambda X. M[X \leftarrow N])$

$(\lambda X. M)[Y \leftarrow N] = \cancel{(\lambda X. M[Y \leftarrow N])}$

$(\lambda Z. M[X \leftarrow Z][Y \leftarrow N])$ $Z \notin FV(M)$

4-2 Function arguments can only be <u>values</u>

$V, U, W = b$
$\mid (\lambda x. M)$
$\mid X$

$f(x) = 42*x + 7$
$f(3+4) =$
math $42 * (3+4) + 7$
ISWIM $f(3+4) = f(7)$
$= 42 * 7 + 7$

β_v (beta v) : Rel (M , $\cancel{N}$)

$((\lambda x. M)\ v)\ \beta_v\ M[X \leftarrow V]$

Δ : Rel (M , M) = Set (M , $\cancel{N}$)

$(o^n\ b_1\ \ldots\ b_n)\ \Delta\ V$ = { (M , N) }
if $\delta(o^n, b_1, \ldots, b_n) = V$

$\delta(\text{inc}, \ulcorner n \urcorner) = \ulcorner n+1 \urcorner$
$\delta(+, \ulcorner n \urcorner, \ulcorner m \urcorner) = \ulcorner n+m \urcorner$ }

$((+\ 5\ 6), 11) \in \Delta$

$\delta(\text{iszero}, 0) = \lambda x. \lambda y. x$
$\delta(\text{iszero}, \ulcorner n+1 \urcorner) = \lambda x. \lambda y. y$

$(\text{if0}\ L\ M\ N) = ((((\text{iszero}\ L)\ (\lambda x. M))\ (\lambda x. N))\ \ulcorner 0 \urcorner)$

$v = \beta_v \cup \Delta$
$\rightarrow_v$ (compatible closure)
$\twoheadrightarrow_v$ refl-trans closure of $\rightarrow_v$
$=_v$ sym closure of $\twoheadrightarrow_v$

α (rename vars) $(\lambda x. x)\ \alpha\ (\lambda y. y)$ (NOT in ISWIM)
η (eta) $(\lambda x. M\ x)\ \eta\ M$
if $X \notin FV(M)$

$\text{eval}_v : M \longrightarrow \text{answers} = A$

$$\text{eval}_v(M) = \begin{cases} b & \text{if } M =_v b \\ \text{'fun} & \text{if } M =_v \lambda X . N \end{cases}$$

$\cancel{X}$

answers = V (constant, function, or variable)

b $(\lambda X . M) \rightarrow$ 'function $X \rightarrow X \in FV(M)$

$A = B \cup \{\text{'fun}\}$

Is eval_v total or partial?

partial:

$M =_v X$

$M =_v (b\ M)$

$=_v (o^n\ b \ldots (\lambda X . N)\ b' \ldots)$

"stuck" (code for "error")

never finish = "diverge" (infinite loop)

$\Omega = ((\lambda x . (x\ x))\ \underline{(\lambda y . (y\ y))})$ $M \ \beta_v\ M$

β_v

$((\lambda y . (y\ y))\ (\lambda y . (y\ y)))$

Y = the Y combinator (or fixed-point operator)

$$Y_v = (\lambda f . (\lambda x . ((\lambda g . (f\ (\lambda x . ((g\ g)\ x))))\ (\lambda g . (f\ (\lambda x . ((g\ g)\ x)))) x)))$$

Theorem: If $K = \lambda Z . \lambda X . L$ then $(K\ (Y_v\ K)) =_v (Y_v\ K)$

$Y_v\ K \Rightarrow_v \ldots [f \leftarrow K] = V$

$Y_v\ K \Rightarrow V \rightarrow_v \lambda x . ((K\ V)\ x)$

$= \lambda x . (([\lambda Z . \lambda X . L]\ V)\ x)$

$\twoheadrightarrow_v \lambda x . L[Z \leftarrow V][X \leftarrow x]$

$= \lambda X . L[Z \leftarrow V]$

$\leftarrow_v ((\lambda Z . \lambda X . L)\ V)$

$= (K\ V)$

$\leftarrow_v (K\ (Y_v\ K))$

4-4 If $M =_v V$, $\forall X, N$,

$$((\lambda X. N)\ M) =_v N[X \leftarrow M] \quad (\beta)$$

justifies inlining (pg 51)