24-1

$T_1 = T_1 \to T_2$ (shows up mainly in Ω).
doesn't exists
$\Rightarrow$ can't have Ω, if, pair

C

```
Type** T1 = NULL;
Type *res = new Arrow (T1, T2)
*T1 = res;
```

List (of numbers) is either + and ×
- an empty
- or, node of a number and List

$$List = Bool + (Num \times List)$$

---

recursive types $T = Num \mid Bool$
$\mid (T+T) \mid (T \times T)$
$\mid (T \to T) \mid A \mid \forall A.T$
$\mid \mu A.T$

List =
$\mu L. (Bool + (Num \times L))$

Binary Tree =
$\mu B. (Num + (B \times B))$

DescendanTree =
$\mu DT. (Bool + (Num \times (\mu DL. (Bool + (DT \times DL)))))$

$$\frac{\Gamma \vdash f : A_1 \to R \qquad \Gamma \vdash a : A_2 \qquad A_1 = A_2}{\Gamma \vdash (f\ a) : R}$$

$\forall a. (a \to Bool) = \forall b. (b \to Bool)$

$\mu a. (a \to Bool) \stackrel{?}{=} \mu b. (b \to Bool)$

$\lambda x. x = \lambda y. y$

$(\mu a. (a \to num)) = ((\mu a. (a \to num)) \to num)$

4-2/ $\boxed{T = T'}$ $\quad \vdash T \leftrightarrow T'$

$$\frac{\vdash T[A \leftarrow A''] \leftrightarrow T'[A' \leftarrow A'']}{\vdash (\mu A.T) \leftrightarrow (\mu A'.T')}$$ where $A'' \notin FVs(T) \cup FVs(T')$ (deal with Romeo)

$$\vdash Num \leftrightarrow Num \qquad \frac{\vdash D_1 \leftrightarrow D_2 \qquad \vdash R_1 \leftrightarrow R_2}{\vdash (D_1 \to R_1) \leftrightarrow (D_2 \to R_2)}$$

$$\frac{\vdash T[A \leftarrow (\mu A.T)] \leftrightarrow T'}{\vdash (\mu A.T) \leftrightarrow T'} \qquad \frac{\vdash T \leftrightarrow T'[A \leftarrow (\mu A.T')]}{\vdash T \leftrightarrow (\mu A.T')}$$

~~A~~ type & its unfold are equal
= equi-recursive

Contrast to iso-recursive

$(\lambda x : (\mu a.(a \to num)).\ ((unfold\ x)\ x)) \quad = \omega$

$$\frac{\dfrac{x : r \vdash x : r}{x : r \vdash (unfold\ x) : (r \to num)} \qquad x : r \vdash x : r \qquad \vdash r \leftrightarrow r}{}$$

where $r = (\mu a.(a \to num))$

$M = \ldots \mid (fold\ M) \mid (unfold\ M)$
$V = \ldots \mid (fold\ V)$
$E = \ldots \mid (fold\ E) \mid (unfold\ E)$

$E[(unfold\ (fold\ V))] \longmapsto E[V]$

$$\frac{\Gamma \vdash M : T[A \leftarrow (\mu A.T)]}{\Gamma \vdash (fold\ M) : (\mu a.T)} \qquad \frac{\Gamma \vdash M : \mu A.T}{\Gamma \vdash (unfold\ M) : T[A \leftarrow (\mu A.T)]}$$

$char_8\ x = \_\_\_$
$int\ y = (int)_{32}\ x;$
~~x~~ al (x)
r10 (y)
mov~~s~~b~~w~~ %al, %r10
movsbw

---

char * xp = ___
int * yp = (~~char~~ *) xp;
xp = 0x8080
yp = 0x8080

24-3

$$\text{List}_\alpha = \text{Bool} + (\alpha \times \text{List})$$

$\text{null} := \Lambda a.\ (\text{fold}\ (\text{inL}\ \text{false}))_{\tau}$

$\text{cons} := \Lambda a.\ (\lambda v{:}a.\ \lambda l{:}\text{List}\alpha.\ (\text{fold}\ (\text{inR}\ (\text{pair}\ v\ l))))$

$\text{car} = \text{first} :=$

$\Lambda a.\ (\lambda l{:}\text{List}\,\alpha.$
$\quad \text{Match}\ (\text{unfold}\ l)$
$\quad (\lambda n{:}\text{bool}.\ \Omega_\alpha)$
$\quad (\lambda p{:}(\alpha \times \text{List}\alpha).\ \text{fst}\ p))$

fold goes after constructors
unfold goes before accessors

$\text{List}\langle A\rangle = \text{Bool} + (A \times \text{List}\langle A\rangle)$

Unit Type : $T = 1$ $\quad V = tt$ or $()$

Bottom Type : $T = 0$ $\quad V =$ ______

$\text{List}\langle A\rangle = 1 + (A \times \text{List}\langle A\rangle)$ $\qquad \delta xy = (\delta x)y + x(\delta y)$

$\delta_A \text{List}\langle A\rangle = \delta_A (1 + (A \times \text{List}\langle A\rangle))$
$= \delta_A 1 + \delta_A (A \times \text{List}\langle A\rangle)$
$= 0 + ((\delta_A A) \times \text{List}\langle A\rangle) + (A \times \delta_A \text{List}\langle A\rangle)$
$= 0 + (1 \times \text{List}\langle A\rangle) + (A \times \delta_A \text{List}\langle A\rangle)$

$\text{Zipper}\langle\text{List}\langle A\rangle\rangle \simeq \text{List}\langle A\rangle + (A \times \delta_A \text{List}\langle A\rangle)$

The quick brown fox | I | jumped over
the lazy student.