23-1

$(\lambda x.\ x+5)$ ISWIM $(5\ \ 7)$

$(\lambda x : \underbrace{Num} .\ x+5)$ Typed-ISWIM

Type Inference : ISWIM $\rightarrow$ Poly-ISWIM (partial)

- add type annotations to untyped terms
  - if they exists
- most polymorphic type

$(\lambda x . x) \Rightarrow \Lambda A . \lambda x : A . x$

$\not\Rightarrow \lambda x : num . x$

normal fun ~~app~~ def:

$$\frac{\Gamma[x \to T] \vdash M : T'}{\Gamma \vdash (\lambda x : T . M) : T \to T'}$$

$I \vdash I : O$

$O \vdash I : O$

inferring rule

$$\frac{\Gamma[x \to T] \vdash M : T'}{\Gamma \vdash (\lambda x . M) : T \to T'}$$

$I \vdash I : O \mid O ; O$

$\Gamma \vdash M : T \mid C ; \mathcal{X}$ ← variables

↑ constraints

constraint generation constraint solving

type inference

C = a set of "$T = T$" equations

$\mathcal{X}$ = a set of A type variables

23-2

$$\frac{\Gamma, X:A \vdash M : T' \mid C ; \mathcal{X} \qquad A \text{ is fresh}}{\Gamma \vdash (\lambda X. M) : (A \to T') \mid C ; \mathcal{X} \cup \{A\}}$$

$T = A \mid \forall A. T$
$T \to T \mid num$

$T = T$
$T_x = T_y \to T_z$

$$\frac{\Gamma \vdash M : T_1 \mid C_1 ; \mathcal{X}_1 \qquad \Gamma \vdash N : T_2 \mid C_2 ; \mathcal{X}_2}{\Gamma \vdash (M\ N) : A \mid C_1 \cup C_2 \cup \{T_1 = T_2 \to A\} ; \mathcal{X}_1 \cup \mathcal{X}_2 \cup \{A\}}$$

$A = \cancel{T_1} \cup T_2$

$\Gamma \vdash num : Num \mid \{\} ; \{\}$ $\qquad$ $\Gamma \vdash x : \Gamma(x) \mid \emptyset ; \emptyset$

$$\frac{\emptyset \vdash 5 : Num \mid \emptyset ; \emptyset \qquad \emptyset \vdash 7 : Num \mid \emptyset ; \emptyset}{\emptyset \vdash (5\ 7) : A \mid \emptyset \cup \emptyset \cup \{Num = Num \to A\} ; \emptyset \cup \emptyset \cup \{A\}}$$

$A \qquad \{Num = Num \to A\} \qquad \{A\}$

$\boxed{2x = 4x}$ $\qquad$ $\boxed{\begin{matrix} x = 7 \\ 2x = 13 \end{matrix}}$

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$$\frac{\Gamma \vdash M : T_1 \mid C_1 ; \mathcal{X}_1 \qquad \Gamma \vdash N : T_2 \mid C_2 ; \mathcal{X}_2}{\Gamma \vdash (M + N) : Num \mid C_1 \cup C_2 \cup \{T_1 = Num\} \cup \{T_2 = Num\} ; \mathcal{X}_1 \cup \mathcal{X}_2}$$

$$\frac{\dfrac{\emptyset, x : A_2 \vdash x : A_2 \mid \emptyset ; \emptyset}{\emptyset \vdash (\lambda x. x) : A_2 \to A_2 \mid \emptyset ; \{A_2\}} \qquad \emptyset \vdash 5 : Num \mid \emptyset ; \emptyset}{\emptyset \vdash ((\lambda x. x)\ 5) : A_1 \mid \{A_2 \to A_2 = Num \to A_1\} ; \{A_1, A_2\}}$$

$C = \boxed{A_2 \to A_2 = Num \to A_1}$

$\mathcal{X} = \boxed{A_1, A_2}$

Type of Program $= \boxed{A_1}$

$C' = \boxed{A_2 = Num}, \boxed{A_2 = A_1}$

$C'' = \boxed{A_2 = Num}, \boxed{Num = A_1}$

$x + y = 16$
$x + 3y = 8$

$x = 16 - y$
$x + 3y = 8$

$x = 16 - y$
$(16 - y) + 3y = 8$

$x = 16 - y$
$y = \frac{8 - 16}{2} = -4$

$x = 20 \qquad y = -4$

$N = N$
$N = \Sigma\, m$
$m = N \cdot x^i$
x, y

Gaussian Elimination

n^3

23-3

$\underline{U}$nification

$U : C \Rightarrow \{A = T\}$

! $U(\{A=T\} \cup C) = U(C[A \leftarrow T]), A = T \quad A \notin FV(T)$

!! $U(\{T=A\} \cup C) = U(\{A=T\} \cup C)$

! $U(\{T_1 \to T_2 = T_3 \to T_4\} \cup C)$

$= U(C \cup \{T_1 = T_3\} \cup \{T_2 = T_4\})$

!! $U(\{T = T\} \cup C) = U(C)$

$U(\emptyset) = \varepsilon$

$$\frac{\emptyset \vdash (\lambda x.x) : A \to A \mid \cancel{\emptyset}\ \{\} ; \{A\}}{\emptyset, x:A \vdash x : A \mid \emptyset ; \emptyset} \qquad x+y = 10$$

$c = \emptyset \quad \mathcal{X} = \{A\} \quad \text{Type} = A \to A$

$\Omega A \ldots$

$\Omega = (\lambda x.x\,x)(\lambda x.x\,x) \qquad \omega \sim (\lambda x.x\,x)$

$$\cfrac{\emptyset \vdash (\lambda x.x\,x) : (A_1 \to A_2) \mid \{A_1 = A_1 \to A_2\} ; \{A_1, A_2\}}{\cfrac{\emptyset, x:A_1 \vdash x\,x : A_2 \mid \{A_1 = A_1 \to A_2\} ; \{A_2\}}{\emptyset, x:A_1 \vdash A_1 \mid \emptyset ; \emptyset \qquad \emptyset, x:A_1 \vdash A_1 \mid \emptyset ; \emptyset}}$$

$C = \{A_1 = A_1 \to A_2\} \quad \mathcal{X} = \{A_1, A_2\} \quad T = A_1 \to A_2$

$C' \ \{A_3 = A_1 \to A_4\}$

$C'[A_1 \leftarrow A_1 \to A_2]$

$= \{A_3 = (A_1 \to A_2) \to A_4$

3-4

let id = $^{0}\lambda x.x$ in
^{4}(if ^{1}(id true)
^{2}(id 5)
^{3}(id 6))

$C = \{ 0 = A \to A$
$1 = Bool$
$0 = Bool \to Bool$
$0 = num \to num \}$

$\Downarrow$

$C' = \{ A = Bool$
$A = Num \}$

$\Downarrow$

$\{ Bool = Num \}$

solution: duplicate id to generate unique constraints on each

let-based polymorphism

$$\frac{\Gamma \vdash N[X \leftarrow M] : T \mid C ; \mathcal{X}}{\Gamma \vdash \text{let } X = M \text{ in } N : T \mid C ; \mathcal{X}}$$