

1-3

Why semantics?

- prove that two programs do the same thing
- one program matches math

context [your program / fun]

$$B_{\text{com}} [x + 2] = B_{\text{com}} [x \ll 1]$$

$$\forall b, b' \forall B_L, R. (B_L \bullet b) \ r_b \ (B_L \bullet b') \quad [\text{context, left}]$$

if $b \ r_b \ b'$

~~(B_R \bullet b) \ r_b \ (B_R \bullet b')~~

$$(b \bullet B_R) \ r_b \ (b' \bullet B_R) \quad [\text{context, right}]$$

if $b \ r_b \ b'$

context-compatible closure of a relation

- context-compat of the single-step
- refl + transitive closure of →
- = symmetry of →→

$$\text{eval}_- (x : \text{stx}) = \left\{ \begin{array}{l} y \text{ s.t. } x \rightarrow\!\!\rightarrow y \\ \text{and } y \text{ is a result} \end{array} \right.$$

a result is a subset of stx

$$B = T \mid F \mid (B \bullet B)$$

$$R = T \mid F$$

2-1 interpreter : programs in A that tell you what B programs do

small-step : simulate B in math

$$p \rightarrow p' \rightarrow \dots \rightarrow \text{ans}$$

big-step : defines in math the meaning

$$p \Downarrow \text{ans} \quad (p \Rightarrow \text{ans})$$

denotational : compiler (in math) from B to math

$$c(p) = \text{math-thing}$$

1) employ professors 2) answers qs for programmers

q₁ : what can programs do? (what's possible)

q₂ : did I/we do the right thing?

└ p eval(p) = ans is ans right?

└ will any program give an ans? always?

└ the same ans?



Deterministic : $p \Rightarrow v \Rightarrow$ there's exactly one path from p to v

$$\text{Function : } \mathbb{R} \left[\text{eval}_r(B_0) = R_1 \wedge \text{eval}_r(B_0) = R_2 \right] \\ \Rightarrow R_1 = R_2$$

$$\text{eval}_r(B) = \begin{cases} F & \text{if } B =_r F \\ T & \text{if } B =_r T \end{cases}$$

$$M = (1+1) + (2+2)$$

$$L = 2 + (2+2) \quad (1+1) + 4$$

$$N = 2+4$$



$$\forall B_0, R_1, R_2 \left[B_0 =_r R_1 \wedge B_0 =_r R_2 \right] \\ \Rightarrow R_1 = R_2$$

$$M =_r N \iff \begin{cases} 1. M = N \text{ (recl)} \end{cases}$$

$$\begin{cases} 2. N =_r M \text{ (sym)} \end{cases}$$

$$\begin{cases} 3. M \rightarrow N \text{ (s.s.)} \end{cases}$$

$$\begin{cases} 4. (L, M =_r L, L =_r N) \text{ (trans)} \end{cases}$$

2-2

$$B_0 \rightarrow R_1$$

$$B_0 \rightarrow R_2$$

$$R_i \in \{F, T\}$$

$$R_1 \rightarrow x, x = R_1 \text{ (only re R1)}$$

~~Church-Rosser Property:~~ Consistency:

$\forall M, N$, If $M =_r N$, exists L , $M \rightarrow_r L$
proof, and $N \rightarrow_r L$.

Induction on $M =_r N$.

(sis) case $M \rightarrow_r N$, $L = N$.

(sym) case $M =_r N$ because $N =_r M$,

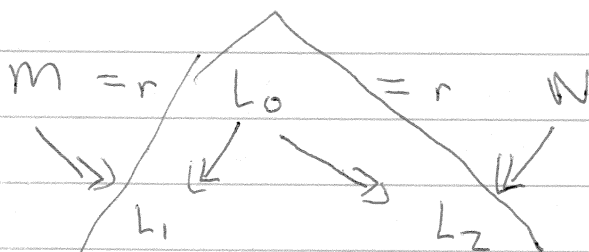
IH: (exists L , $N \rightarrow_r L$ and $M \rightarrow_r L$)
exact IH.

(refl) case $M =_r N$ because $M = N$, $L = N$.

(trans) case $M =_r N$ because $M =_r L_0$ and $L_0 =_r N$,

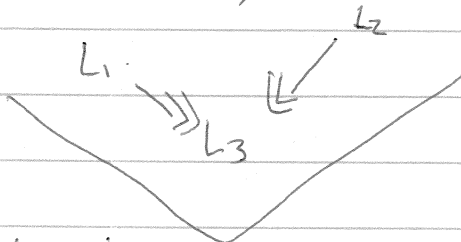
IH₁: ($\exists L_1$, $M \rightarrow_r L_1$ and $L_0 \rightarrow_r L_1$)

IH₂: ($\exists L_2$, $L_0 \rightarrow_r L_2$ and $N \rightarrow_r L_2$)



apply Diamond Property

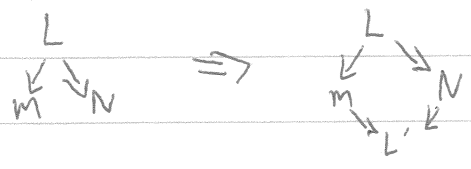
\Rightarrow



ans is L_3 !

2-3 Diamond property called Church-Rosser Property

If $L \Rightarrow_r M$ and $L \Rightarrow_r N$, then $\exists L'$
 $M \Rightarrow_r L'$ and $N \Rightarrow_r L'$



$$X \Rightarrow_r Y = X r Y \text{ (base)}$$

$$X = Y \text{ (refl)}$$

$$\exists z. X \Rightarrow_r z \text{ and } z \Rightarrow_r Y \text{ (trans)}$$

$$\frac{b \Rightarrow_r b'}{(B \cdot b) \Rightarrow_r (B \cdot b')}$$

← comput, left
 ← comput, right

case refl, $L' = M$
 case base. ~~if~~

$L r M$ and $L r N$
 \rightarrow contradiction!

case trans.

case comput, left:

$$L = (B_m \cdot b_m) \quad b_m \Rightarrow_r b'_m \quad L = (B_n \cdot b_n) \quad b_n \Rightarrow_r b'_n$$

$$M = (B_m \cdot b'_m) \quad N = (B_n \cdot b'_n)$$

$$B_m = B_n \quad b_m = b_n$$

$$b \Rightarrow_r b'_m \quad \wedge \quad b \Rightarrow_r b'_n$$

$$\text{IH: } \exists b_1, b'_m \Rightarrow_r b_1 \quad \wedge \quad b'_n \Rightarrow_r b_1$$

case comput, right:
 similar

$$M =_r R \text{ iff } M \Rightarrow_r R$$

↓
 $\Rightarrow_r + \text{sym}$

sym doesn't matter

$\forall B_0 \in B. \exists R_0 \in R. \text{eval}(B_0) = R_0.$ $\vdash \forall R_0. \text{eval}(B_0) \neq R_0$

interp: $B \Rightarrow R$ \rightarrow proof \rightarrow normal math

\rightarrow constructive

\hookrightarrow give a witness