18-1

```
int n = 0;
for (int i=0; i < 17; i++) {
  n = n + i;
}
return n;
```

constant space

⇒ 319

ISWIM

319

(+ (+ .... (+ 0 1) 2) 3) ...) 16)

```
(define (sum i n)
  (if (= i 17)
      n
      (sum (add1 i) (+ i n))))
(sum 0 0)
```

constant space

i' = i + 1
n' = i + n

```
(define (osum n)
  (if (zero? n)
      0
      (+ (osum (sub1 n))
         n)))
(osum 17)
```

linear space

E[if true M N] → E[M]
E[if false M N] → E[N]

(sum 0 0)
⇒ (if (= 0 17) 0 (sum (add1 0) (+ 0 0)))
→ false
→³ (sum 1 0)
⇒ (if (= 1 17) 0 (sum (add1 1) (+ 1 0)))
→⁴ (sum 2 1)

```
for (int i=0; i < 17; i++) {
  0 = 0 + i;
}
return 0;
```

8-2/1.
```
1. int n = 0;
2. int i = 0;
3. while (i < 17) {
4.     n = i + n;
5.     i = i + 1;
6. }
7. ret. n;
```

#line	mem
①	[]
⇒ ②	[n ↦ 0]
⇒ ③	[n ↦ 0, i ↦ 0]
⇒ ④	
⇒ ⑤	[n ↦ 0, i ↦ 0]
⇒ ⑥	[n ↦ 0, i ↦ 1]
⇒ ③	
⇒* ③	[n ↦ 319, i ↦ 17]
⇒ ⑦	

State-ISWIM

M = ... | (set! X M)

([λX. ((λY. X) (set! X (+ X 1)))] 12)

⇒ ((λY. X) (set! X (+ X 1))) [X ↦ 12]

⇒ (λY. X) void) [X ↦ 13]

⇒ X [X ↦ 13, Y ↦ void]

⇒ 13

Normal

E = [] | (E M) | (V E)

E[((λX. M)V)] ⟶ E[M[X ← V]]

C

```
int n=0;
for (int i=0; i<17; i++)
    n = i + n;
return n+7;
```

ISWIM

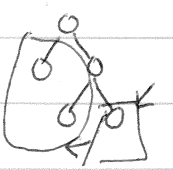
((λN. (+ n 7)) 0)

Idea #1

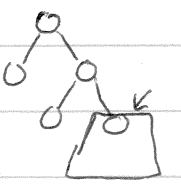
E = ... | ((λX. E) V)

E[X] ⟶ V if E = ...((λX. E')V)

E[((λX. E'[X]) V)] ⟶ E[((λX. E'[V]) V)] E' can't mention X



exponential



linear

Idea #2

E[((λX. M) V)] (letrec ([X0 V0] ... [Xn Vn]) E[((λX. M) V)])

⟶ (letrec ([X0 V0] ... [Xn Vn] [X V]) E[M])

18-3/ Idea: Add pointers + memory

Pointer = σ

Σ = Memory = $\sigma \to V$

$m = \ldots \mid \sigma \mid (\text{set! } \sigma\ m) \mid \text{void}$

$V = \ldots \mid \text{void}$

~~Program = M (Program $\mapsto$ Program)~~

Program = $\langle M, \Sigma \rangle$ (Prog $\mapsto$ Prog)

eval(m) = $\langle m, \emptyset \rangle \longmapsto^* \langle V, \Sigma \rangle$

if $V = \lambda X$, Fun

$V = \text{void}$, void

$V = b$, b

$\langle E[\sigma], \Sigma \rangle \longmapsto \langle E[\Sigma(\sigma)], \Sigma \rangle$

$\langle E[(\text{set! } \sigma\ V)], \Sigma \rangle \mapsto \langle E[\text{void}], \Sigma[\sigma \mapsto V] \rangle$

$\langle E[((\lambda X.\ m)\ V)], \Sigma \rangle$

$\mapsto \langle E[m[x \leftarrow \sigma]], \Sigma[\sigma \mapsto V] \rangle$

$\sigma \notin \text{dom}(\Sigma)$

$\langle ((\lambda X.\ ((\lambda Y.\ X)\ (\text{set! } X\ (+\ X\ 1))))\ 12),\ \emptyset \rangle$

$\to \langle ((\lambda Y.\ x92)\ (\text{set! } x92\ (+\ x92\ 1))),\ [x92 \mapsto 12] \rangle$

$\to$ F[12]

$\to \langle ((\lambda Y.\ x92)\ (\text{set! } x92\ [13])),\ [x92 \mapsto 12] \rangle$

$\to \langle ((\lambda Y.\ x92)\ \text{void}),\ [x92 \mapsto 13] \rangle$

$\to \langle x92,\ [x92 \mapsto 12,\ x88 \mapsto \text{void}] \rangle$

$\to \langle 12,\ [x92 \mapsto 12,\ x88 \mapsto \text{void}] \rangle$

18-4/ let $X = M$ in N

$:=$

$((\lambda X.N)\ M)$

begin M N

$:=$

$((\lambda X.N)\ M)$

$X \notin FV(N)$

let count =

(let $c = 0$ in

$(\lambda X.$ (begin (set! c $(+\ c\ x)$)

c))) , $\emptyset$ >

in

(+ (count 2)

(count 4))

$\Rightarrow$

let count =

: $\lambda X.$ (begin (set! σ_0 $(+\ \sigma_0\ x)$)

σ_0) , $[\sigma_0 \mapsto 0]$ >

in

(+ (count 2) (count 4))

$\Rightarrow^*$

: (+ (begin (set! σ_0 $(+\ \sigma_0\ \sigma_1)$) σ_0) (begin (set! σ_0 $(+\ \sigma_0\ \sigma_2)$) σ_0)) , $[\sigma_0 \mapsto 0, \sigma_1 \mapsto 2, \sigma_2 \mapsto 4]$ >

$\Rightarrow^*$

: (+ 2 (begin (set! σ_0 $(+\ \sigma_0\ \sigma_2)$) σ_0)) , $[\sigma_0 \mapsto 2, \sigma_1 \mapsto 2, \sigma_2 \mapsto 4]$ >

$\Rightarrow^*$

: (+ 2 6) , $[\sigma_0 \mapsto 6, \sigma_1 \mapsto 2, \sigma_2 \mapsto 4]$ >

$\Rightarrow$

: 8 , " >