1-1

interest

- you trust bank trusts math
- who watches mathematicians?
- what about compilers?
- $x+y = y+x \qquad x+0 = x$

  $x+(y+z) = (x+y)+z$

Syntax

In C, "x = y++;" is a statement

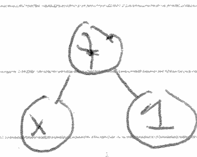
expr ⊆ statement ∈ fun ∈ programs

A. "return x+1;" - C

B. "return x+1;" - Java

env/context "x is a int"

semantics "what + and ints mean"

" (+ x 1) = 

Clojure, Racket, Lisp, Scheme

Semantics

a relation on syntax ⇒ "is"

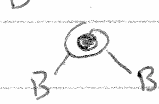
(stx, stx) set of trees

"1 plus 1 is 2"

(+ 1 1) $\overset{=}{R}$ 2

Context-Free Grammar (Foundations

$B = T \mid F \mid (B \bullet B)$;; BNF

T F 

Backus-Naur Form

$((F \bullet F) \bullet T) \in B$?

Jay $\notin$ B

1-2

Relations

- a set of pairs of X and Y (Rel over X & Y)
  - $X = Y = stx$ (stx, stx)
- "a R b" to mean "$(a,b) \in R$" "a is related by R to b"
- "$(F \bullet F) = F$"
- equivalence has 3 - propes
  - reflexive : $\forall a,\ a\ R\ a$
  - symmetry : $\forall x, y,\ x\ R\ y \Rightarrow y\ R\ x$
  - transitive : $\forall x, y, z,\ x\ R\ y \wedge y\ R\ z \Rightarrow x\ R\ z$

$(1+1) = 2 \quad 2 = 2+0 \quad (1+1) = 2+0$

$r_B : Rel(B, B)$

$\forall b,\ (F \bullet b)\ r_B\ b$ [f]

$\forall b,\ (T \bullet b)\ r_B\ T$ [t]

"small-step relation"

(1) B means r_B

(2) B means what this C prog. does

= "an interpreter"

$(F \bullet (F \bullet (T \bullet T)))$ [f] $(F \bullet (T \bullet T))$ [f] $(T \bullet T)$ [t] T

$(1 + (1 + (1 + 1))) = (1 + (1 + 2)) = (1 + 3) = 4$

$\forall b,\ b\ r_B\ b$ [refl]

$\forall b_1, b_2,\ b_1\ r_B\ b_2 \Rightarrow b_2\ r_B\ b_1$ [sym]

reflexivate (r_B) = r_B but reflexive

"taking the reflexive closure of r_B"

" transitive closure " rts rb

" syme "

$(F \bullet (F \bullet (T \bullet T)))\ rtsr_b\ T$ "big step" semantics

why semantics?
- prove that two programs do the same thing
- one program matches math

context [your program / fun]

B.com [x + 2] = B.com [x << 1]

$\forall b, b' \forall B_L, R.$ $(B_L \bullet b) \; r_b \; (B_L \bullet b')$ [context, left]
if $b \; r_b \; b'$

~~$(B_R \bullet$~~

$(b \bullet B_r) \; r_b \; (b' \bullet B_r)$ [context, right]
if $b \; r_b \; b'$

context-compatible closure of a relation

$\rightarrow_{\_}$ context compat of the single-step
$\twoheadrightarrow_{\_}$ refl + transitive closure of $\rightarrow_{\_}$
$=_{\_}$ symmetry of $\twoheadrightarrow_{\_}$

$$\text{eval}_{\_}(x : \text{stx}) = \left\{ y \;\; \text{s.t. } x \twoheadrightarrow y \text{ and } y \text{ is a result} \right.$$

a result is a subset of stx

$B = T \mid F \mid (B \bullet B)$

$R = T \mid F$