CK-machine —— $\langle M, K \rangle$

control (code) continuation (context)

$$K = \text{ret} \mid \text{fun}(N, K) \mid \text{arg}(V, K)$$

$\forall X, K.$

$\langle X, K \rangle \not\mapsto$

$V = b \mid \lambda X. M$

$$\langle (M\ N), K \rangle \mapsto \langle M, \text{fun}(N, K) \rangle$$
$$\langle V, \text{fun}(N, K) \rangle \mapsto \langle N, \text{arg}(V, K) \rangle$$
$$\langle V, \text{arg}(\lambda X. M, K) \rangle \mapsto \langle M[X \leftarrow V], K \rangle$$

$$M = (((\ (x\ x)\ (x\ x))\ (\ (x\ x)\ (x\ x)))\ ((\ (x\ x)\ (x\ x))\ (\ (x\ x)\ (x\ x))))$$

$$X[X \leftarrow V] = V$$
$$Y[X \leftarrow V] = Y$$
$$(M\ N)[X \leftarrow V] = (M[X \leftarrow V]\ \ N[X \leftarrow V])$$
$$(\lambda X. M)[X \leftarrow V] = (\lambda X. M)$$
$$(\lambda Y. M)[X \leftarrow V] = (\lambda Y. M[X \leftarrow V])$$

9-2

CEK-machine $\langle C, \mathcal{E}, k \rangle$

K → continuation

C → code

E → Environment — a list of substitutions to perform later

$$\langle X, \mathcal{E}, k \rangle \mapsto \langle \mathcal{E}(X), \mathcal{E}, k \rangle$$

$$\mathcal{E} = mt \mid \mathcal{E}[X \leftarrow V]$$

$$\mathcal{E}[X \leftarrow V](X) = V$$

$$\mathcal{E}[Y \leftarrow V](X) = \mathcal{E}(X)$$

$$mt(X) = \perp \text{ (error)}$$

```
interface Env { Value lookup (Variable X); }
class Mt imp Env {
  Value lookup (Var X) { error; } }
class Some imp Env {
   Env E1 Var Y1 Value V1
   Value lookup (Var X) {
     if (X == Y) { ret V; }
     else { ret E1.lookup(X); } } }
```

$$\langle V, \mathcal{E}, arg(\lambda X.m, k) \rangle$$

$$\mapsto \langle m, \underbrace{\mathcal{E}[X \leftarrow V]}_{\text{new Some}(\mathcal{E}, X, V)}, k \rangle$$

$$\langle (M\ N), \mathcal{E}, k \rangle \mapsto \langle M, \mathcal{E}, fun(N, k) \rangle$$

$$\langle V, \mathcal{E}, fun(N, k) \rangle \mapsto \langle N, \mathcal{E}, arg(V, k) \rangle$$

9-3/ $\langle ((\lambda x.\ 3 + x)\ 7),\ mt,\ ret \rangle$

$\langle (\lambda x. 3+x),\ mt,\ fun(7, ret) \rangle$

$\langle 7,\ mt,\ arg(\lambda x. 3+x,\ ret) \rangle$

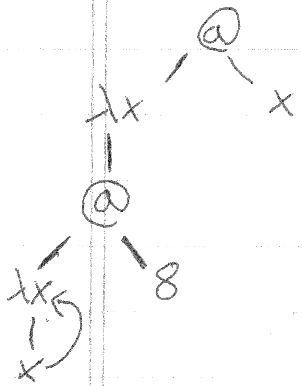
$\langle 3+x,\ mt[x \leftarrow 7],\ ret \rangle$

$\langle x,\ mt[x \leftarrow 7],\ prim(+, 3, \perp, ret) \rangle$

$\langle 7,\ mt[x \leftarrow 7],\ pr$ '' $\rangle$

$\langle 10,\ mt[x \leftarrow 7],\ ret \rangle$

$((\lambda x.\ ((\lambda x.x)\ 8))\ \cancel{4}x) = M_0$



$\langle M_0,\ mt,\ ret \rangle$

$\langle ((\lambda x.x)\ 8),\ mt,\ fun(x, ret) \rangle$

$\langle (\lambda x.x),\ mt,\ fun(8, fun(x, ret)) \rangle$

$\langle 8,\ mt,\ arg(\lambda x.x,\ fun(x, ret)) \rangle$

$\langle x,\ mt[x \leftarrow 8],\ fun(x, ret) \rangle$

$\langle 8,\ mt[x \leftarrow 8],\ fun(x, ret) \rangle$

$\langle x,\ mt[x \leftarrow 8],\ arg(8, ret) \rangle$

$\langle 8,\ mt[x \leftarrow 8],\ arg(8, ret) \rangle$

$\langle V,\ \mathcal{E},\ fun(N, k) \rangle$

$\quad \mapsto \langle N,\ \mathcal{E},\ arg(V, k) \rangle$

((fun pos,) [X ← V]
(arg pos) [X ← V])

((λx. ␣) 8)

9-4

$K' = \text{ret}$
$\quad \mid \text{fun}(N, \mathcal{E}, k)$
$\quad \mid \text{arg}(V, k)$

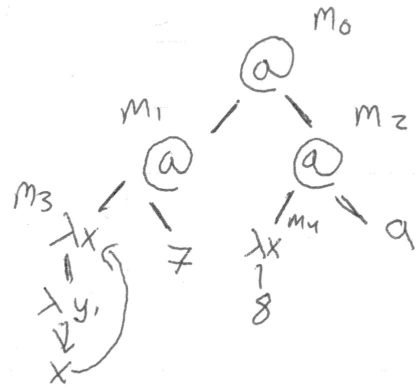
$\langle (M\ N), \mathcal{E}, k \rangle$
$\quad \longmapsto \langle M, \mathcal{E}, \text{fun}(N, \mathcal{E}, k) \rangle$

$\langle V, \mathcal{E}, \text{fun}(N, \mathcal{E}', k) \rangle$
$\quad \longmapsto \langle N, \mathcal{E}', \text{arg}(V, k) \rangle$
$\quad\quad \text{arg}(V, \mathcal{E}, k)$

$((\lambda x. (((\lambda x. (\lambda y. x)) 7)\ 8))\ 9)$

$\langle V, \mathcal{E}, \text{arg}(\lambda X. M, k) \rangle$
$\quad \longmapsto \langle M, \mathcal{E}[X \leftarrow V], k \rangle$

$(((\lambda x.$
$\quad (\lambda y. x)\quad)$
$\quad 7)$
$((\lambda x. 8)\ 9))$



$\langle M_0, mt, \text{ret} \rangle$
$\langle M_1, mt, \text{fun}(M_2, mt, \text{ret}) \rangle$
$\langle M_3, mt, \text{fun}(7, mt, \text{fun}(M_2, mt, \text{ret})) \rangle$
$\langle 7, mt, \text{arg}(M_3, mt, \quad '' \quad) \rangle$
$\rightarrow \langle \lambda y. x, mt[x \leftarrow 7], \text{fun}(M_2, mt, \text{ret}) \rangle$
$\rightarrow \langle M_2, mt, \text{arg}(\lambda y. x, \text{ret}) \rangle$
$\langle M_4, mt, \text{fun}(9, mt, \quad '' \quad) \rangle$
$\langle 9, mt, \text{arg}(\lambda x. 8, \text{arg}(\lambda y. x, \text{ret})) \rangle$
$\langle 8, mt[x \leftarrow 9], \text{arg}(\lambda y. x, \text{ret}) \rangle$
$\langle x, mt[x \leftarrow 9][y \leftarrow 8], \text{ret} \rangle$
$\langle 9, \quad '' \quad , \text{ret} \rangle$

9-5 / real CEK-machine

$$
\begin{aligned}
N, M, C &= X \\
&\mid (M\ N) \\
&\mid (\lambda X . M) \\
&\mid b \mid (o^n\ M \ldots) \\
V &= b \\
&\mid \underbrace{\text{clo}}_{\text{closure}}(\lambda X . M, \mathcal{E})
\end{aligned}
\qquad
\begin{aligned}
\mathcal{E} &= \text{mt} \\
&\mid \mathcal{E}[X \leftarrow V]
\end{aligned}
\qquad
\begin{aligned}
K &= \text{ret} \\
&\mid \text{fun}(N, \mathcal{E}, K) \\
&\mid \text{arg}(V, K) \\
&\mid \text{prim}(o^n, V \ldots, M \ldots, K)
\end{aligned}
$$

0 $\langle X, \mathcal{E}, K \rangle \longmapsto \langle \mathcal{E}(X), \text{mt}, K \rangle$

1 $\langle \lambda X . M, \mathcal{E}, K \rangle \longmapsto \langle \text{clo}(\lambda X . M, \mathcal{E}'), \text{mt}, K \rangle$

$\mathcal{E}' = \mathcal{E}$ restricted to $\text{FreeVars}(M)$

2 $\langle (M\ N), \mathcal{E}, K \rangle \longmapsto \langle M, \mathcal{E}, \text{fun}(N, \mathcal{E}, K) \rangle$

3 $\langle V, \mathcal{E}, \text{fun}(N, \mathcal{E}', K) \rangle \longmapsto \langle N, \mathcal{E}', \text{arg}(V, K) \rangle$

4 $\langle V, \mathcal{E}, \text{arg}(\text{clo}(\lambda X . M, \mathcal{E}'), K) \rangle$

$\longmapsto \langle M, \mathcal{E}'[X \leftarrow V], K \rangle$

env impl	copy (2)	add (4)	lookup (0)	restrict
linked-list	0	1	n	n
pure hash	0	lg n	lg n	m lg n
mut hash	n	1	1	n
semi	1	linear in changes	1	? bad
vec	0	0	1	m

doing restriction ⟶ "flat closure"

not ⟶ "linked closure"