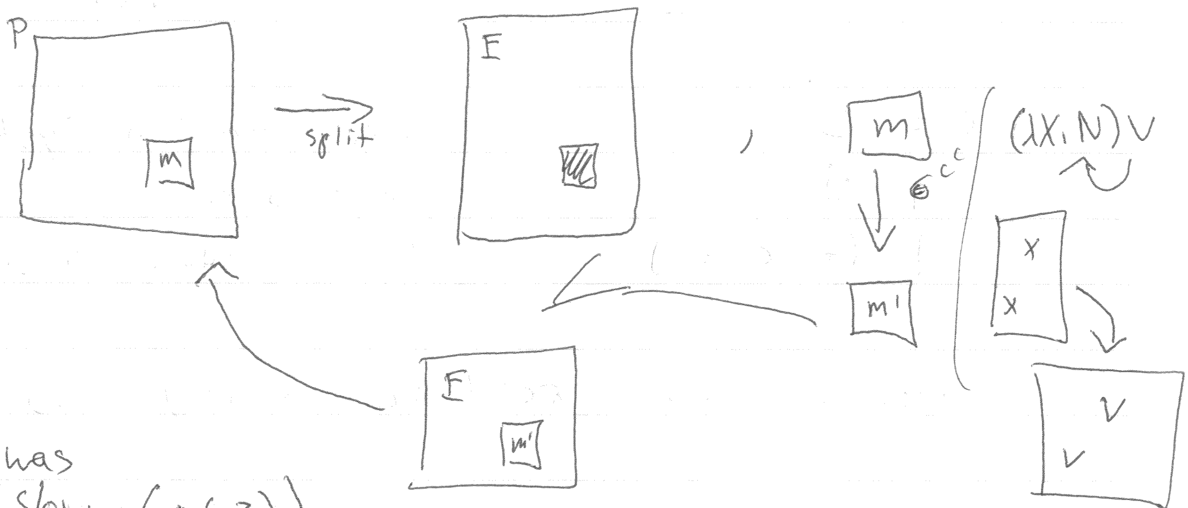
7-1

semantics = "algebra" of programs

$7 + 14 * (56 + 81^{23}) = 6 * 92^3 + 18$

$7 + x * (56 + 81^{23}) = x * 92^3 + 18$



$\mapsto_v$ was slow ($O(n^2)$)

machine semantics = how to run programs

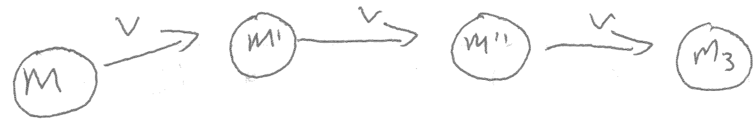
$\rightarrow_v$: program, program

$\rightarrow_v = \mapsto_v$ = fast ($O(1)$)

1. Is it a value?
2. What is E? What is M = (V V) or (o^n V ...)
3. Apply beta (β) or delta (Δ) to M gets M'
4. Produce E[M']
5. Repeat

semantics were always rel(programs)

programs were expression



Not Finite-State because Programs is infinite

e = 0
| 1
| (+ e e)

Labeled Transition System
Kripke structure
Infinite-State Automata

Machines are rel(machine-programs)

CC-machine states = ~~(E x M)~~ (M x E)

M is the Control String
E is the eval Context

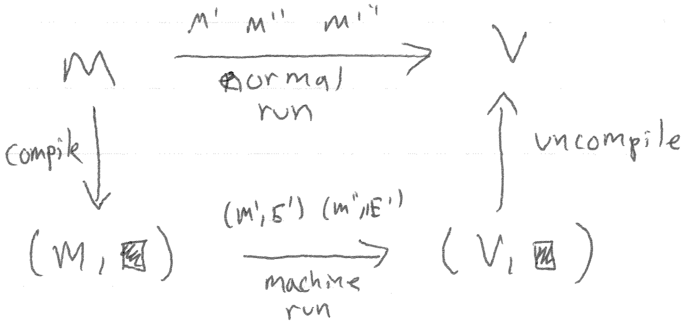
M ↓ ISWIM program
E ↓ Eval Context

compile : program → machine-program

compile (M) = (M, ▩)

uncompile : machine value → program value

uncompile ((V, ▩)) = V



[1] $$\langle (M\ N),\ E \rangle \longmapsto_{cc} \langle M,\ E[(\blacksquare\ N)] \rangle \quad \text{if } M \notin V$$

parsing/split

[2] $$\langle (V\ N),\ E \rangle \longmapsto_{cc} \langle N,\ E[(V\ \blacksquare)] \rangle \quad \text{if } N \notin V$$

[3] work $$\langle ((\lambda X.M)\ V),\ E \rangle \longmapsto_{cc} \langle M[X \leftarrow V],\ E \rangle$$

[4] fill $$\langle V,\ E[(\blacksquare\ N)] \rangle \longmapsto_{cc} \langle (V\ N),\ E \rangle$$

[5] $$\langle V,\ E[(U\ \blacksquare)] \rangle \longmapsto_{cc} \langle (U\ V),\ E \rangle$$

$$\langle (o^n\ V \ldots\ M\ N \ldots),\ E \rangle \longmapsto_{cc} \langle M,\ E[(o^n\ V \ldots\ \blacksquare\ N \ldots)] \rangle \quad M \notin V$$

$$\langle U,\ E[(o^n\ V \ldots\ \blacksquare\ N \ldots)] \rangle \longmapsto_{cc} \langle (o^n\ V \ldots\ U\ N \ldots),\ E \rangle$$

$$\langle (o^n\ V \ldots),\ E \rangle \longmapsto_{cc} \langle \delta(o^n, V \ldots),\ E \rangle$$

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cc: 1 2 1 3 3 3 1 2 3 5 4 4 5 4

sr: 1213454 | 1213454 | 1213454 | 121123544454

~~((V M) N)~~

$$((V'\ (((\lambda X.M)\ V)\ N))\ N') = E[(\lambda X.M)\ V]$$

$$E = ((V'\ (\blacksquare\ N))\ N') \qquad \Downarrow \quad E[(M[X \leftarrow V]]$$

$\text{eval}_{cc}(M) =$

if $\langle M, \blacksquare \rangle \mapsto\!\!\!\to_{cc} \langle V, \blacksquare \rangle$

then if V is a b, then ret b

o.w. ret 'fun

Theorem: $\text{eval}_{cc} = \text{eval}_v^s$

$\iff \forall M, V,\quad M \mapsto\!\!\!\to_v V$ iff $\langle M, \blacksquare \rangle \mapsto\!\!\!\to_{cc} \langle V, \blacksquare \rangle$

generalize. $\forall M, E, V,\quad E[M] \mapsto\!\!\!\to_v E[V]$

iff $\langle M, E \rangle \mapsto\!\!\!\to_{cc} \langle V, E \rangle$

Lemma: If $M = E'[L]$ and $L \; v \; L'$ then

$\langle M, E \rangle \mapsto\!\!\!\to_{cc} \langle L, E[E'] \rangle$

SCC-machine = Simplified CC-machine

$\mapsto_{scc} : \langle M, E \rangle \to \langle M, E \rangle$

1' $\langle (M\ N), E \rangle \mapsto_{scc} \langle M, E[(\blacksquare\ N)] \rangle$ A

4'+2' $\langle V, E[(\blacksquare\ N)] \rangle \mapsto_{scc} \langle N, E[(V\ \blacksquare)] \rangle$ B

5'+3' $\langle V, E[((\lambda X.M)\ \blacksquare)] \rangle \mapsto_{scc} \langle M[X \leftarrow V], E \rangle$ C

$\langle (o^n\ M\ N \ldots), E \rangle \mapsto_{scc}$ D

$\langle M, E[(o^n\ \blacksquare\ N \ldots)] \rangle$

$\langle V, E[(o^n\ U \ldots\ \blacksquare\ M\ N \ldots)] \rangle \mapsto_{scc}$ E

$\langle M, E[(o^n\ U \ldots\ V\ \blacksquare\ N \ldots)] \rangle$

$\langle V, E[(o^n\ U \ldots\ \blacksquare)] \rangle \mapsto_{scc}$ F

$\langle \delta(o^n, U \ldots V), E \rangle$

7-5

$\langle (+\ ((\lambda x.x)\ 3)\ \ ((\lambda y.y)\ 4)),\ \boxtimes \rangle$

D $\langle ((\lambda x.x)\ 3),\ (+\ \boxtimes\ ((\lambda y.y)\ 4)) \rangle$

A $\langle (\lambda x.x),\ (+\ (\boxtimes\ 3)\ ((\lambda y.y)\ 4)) \rangle$

B $\langle 3,\ (+\ ((\lambda x.x)\ \boxtimes)\ ((\lambda y.y)\ 4)) \rangle$

C $\langle x[x \leftarrow 3],\ (+\ \boxtimes\ ((\lambda y.y)\ 4)) \rangle$

$= 3$

E $\langle ((\lambda y.y)\ 4),\ (+\ 3\ \boxtimes) \rangle$

A

B

C $\langle 4,\ (+\ 3\ \boxtimes) \rangle$

F $\langle \delta(+, 3, 4),\ \boxtimes \rangle$

$= 7$

$\langle 7,\ \boxtimes \rangle$